# FINITE-TIME ANALYSIS OF SINGLE-TIMESCALE ACTOR-CRITIC

Xuyang Chen National University of Singapore chenxuyang@u.nus.edu Lin Zhao National University of Singapore elezhli@nus.edu.sg

# ABSTRACT

Despite the great empirical success of actor-critic methods, its finite-time convergence is still poorly understood in its most practical form. In particular, the analysis of single-timescale actor-critic presents significant challenges due to the highly inaccurate critic estimation and the complex error propagation dynamics over iterations. Existing works on analyzing single-timescale actor-critic only focus on the i.i.d. sampling or tabular setting for simplicity, which is rarely the case in practical applications. We consider the more practical online single-timescale actor-critic algorithm on continuous state space, where the critic is updated with a single Markovian sample per actor step. We prove that the online single-timescale actor-critic method is guaranteed to find an  $\epsilon$ -approximate stationary point with  $\widetilde{\mathcal{O}}(\epsilon^{-2})$  sample complexity under standard assumptions, which can be further improved to  $\mathcal{O}(\epsilon^{-2})$  under i.i.d. sampling. Our analysis develops a novel framework that evaluates and controls the error propagation between actor and critic in a systematic way. To our knowledge, this is the first finite-time analysis for online single-timescale actor-critic method. Overall, our results compare favorably to the existing literature on analyzing actor-critic in terms of considering the most practical settings and requiring weaker assumptions.

# 1 Introduction

Actor-critic (AC) methods have achieved huge success in solving many challenging reinforcement learning (RL) problems [1, 2, 3]. In AC methods, the actor (i.e., the policy) is updated by the estimated policy gradient (PG) which is a function of the Q-value (action-value function) corresponding to this policy. AC methods employ a parallel critic update to bootstrap the Q-value for policy gradient estimation, which often enjoys reduced variance and fast convergence in practical implementations.

Despite the empirical success, the convergence analysis of AC in the most practical single-timescale form remains largely unknown. A large body of existing works consider the double-loop setting. In double-loop AC, the inner loop critic update takes sufficient steps to accurately estimate the Q-value for a given actor from the outer loop [4, 5, 6, 7]. As a result, the analysis of critic can be easily decoupled from that of the actor, with a policy evaluation sub-problem in the inner loop and a perturbed gradient descent in the outer loop. Its finite-time convergence is easy to analyse and well understood in general [5, 4, 8]. Nevertheless, double-loop setting is mainly for the ease of analysis, which is rarely used in practice. In fact, since it requires an accurate critic estimation, it is in general sample inefficient compared to the single-loop variant [9]. Moreover, it's unclear whether an inner loop of accurate policy evaluation is really necessary given that it only corresponds to a transient policy during update.

Another body of works consider the (single-loop) two time-scale algorithm [10, 9, 11], where the actor and the critic are updated simultaneously in each iteration using stepsizes of different timescales. The actor stepsize is typically smaller than that of the critic, with their ratio goes to zero as the iteration number goes to infinity. The two-timescale allows the critic to approximate the correct Q-value in an asymptotic way. This design essentially allows for a decoupled convergence analysis of the actor and the critic. Again, this variant is not very often used in practice and can be sample inefficient as the actor update is artificially slowed down.

In this paper, we consider the most practical single-timescale AC algorithm, which is the one introduced in many literature as well as in [12] as a classic AC algorithm. In single-timescale AC, the stepsizes for the critic and the actor diminishes at the same timescale. Unlike the aforementioned variants, there is no specialized design that helps

simplify the convergence analysis in single-timescale AC. Rather, the error presents in the critic estimation can be substantial, and the close coupling between the parallel critic update and actor update can lead to unstable error propagation. Indeed, it remains unclear under what condition the errors will converge to zero. To study its finite-time convergence, we consider the challenging undiscounted time-average reward formulation [12], which consists of three parallel updates: the (time-average) reward estimator, the critic estimator, and the actor estimator. We keep track of the reward estimation error, the critic error, and the policy gradient norm (which measures the actor error) by deriving an implicit bound for each of them. They are then analyzed altogether as an interconnected system inspired by [13] to establish the convergence simultaneously. Particularly, we identify a threshold of the (constant) ratio between the actor stepsize and the critic stepsize, below which all three errors will diminish to zero, despite the inaccurate estimation in all three updates (reward estimation, critic, actor). Our analysis applies to both i.i.d sampling and online Markovian sampling. To our knowledge, our work presents the first finite-time analysis for online single-timescale AC algorithm, which improves the results of existing works on single-timescale AC [14, 13] by considering Markovian sampling and requiring less assumptions (see details in 1.1).

## 1.1 Main Contributions

We summarise our main contributions as follows:

• We provide the first finite-time analysis for the single-timescale AC under Markovian sampling with  $\tilde{\mathcal{O}}(\epsilon^{-2})$  sample complexity. We further show that this sample complexity can be improved to  $\mathcal{O}(\epsilon^{-2})$  under i.i.d. sampling, which matches the state-of-the-art performance of SGD on general non-convex optimization problem. We remark that the additional logarithmic term hide by  $\tilde{\mathcal{O}}(\cdot)$  under Markovian sampling is caused by the mixing time of the Markov chain.

• Our result outperforms all existing works on single-timescale AC. To our knowledge, the only other results of single-timescale AC in general case are from [14] and [13], both of which obtain a sample complexity of  $O(\epsilon^{-2})$ . However, [14] considered the i.i.d. sampling and their analysis highly relies on the smoothness of stationary distribution which cannot be justified easily. The authors left the removal of this assumption and the extension to Markovian sampling for future research. Both challenges left in [14] are well resolved in our work.

Besides, [13] also assumed i.i.d. sampling and only considered the tabular case, whereas we allow the state space S to be infinite. It is believed in [13] that the i.i.d. sampling is important to guarantee the convergence of single-timescale AC with TD(0) update. However, we show that single-timescale AC with TD(0) update does converge under Markovian sampling.

Moreover, compared to the state-of-the-art two-timescale AC in [10], we generalize their results to the more challenging single-timescale case under exactly the same settings and assumptions, purely through the improvement of our analysis. Beyond that, we are able to improve their sample complexity from  $\tilde{\mathcal{O}}(\epsilon^{-2.5})$  to  $\tilde{\mathcal{O}}(\epsilon^{-2})$ .

• Technically, we develop a novel analysis framework that can establish the finite-time convergence for singletimescale AC under standard assumptions. The existing analysis for double-loop AC [4] and two-timescale AC [10] hinge on decoupling the analysis of actor and critic, which typically establishes the convergence of critic first and then actor [4, 10, 14]. We instead investigate the evolution of the coupled estimation errors of the time-average reward, the critic, and the policy gradient norm altogether as an interconnected system. In particular, we identify a threshold of the ratio between the actor stepsize and the critic stepsize, below which all estimation errors diminishes. This threshold can serve as a guidance for choosing the stepsize in practice to ensure a stable learning. Moreover, our new proof framework can provide insights for finite-time analysis of other single-timescale stochastic approximation algorithms as well.

# 1.2 Related Work

Actor-Critic methods. The first AC algorithm was proposed by [15]. [16] extended it to the natural AC algorithm. The asymptotic convergence of AC algorithms has been well established in [16, 17, 18, 19] under various settings. Many recent works focused on the finite-time convergence of AC methods. Under the double-loop setting, [4] established the global convergence of AC methods for solving linear quadratic regulator (LQR). [6] studied the global convergence of AC methods with both the actor and the critic being parameterized by neural networks. [5] studied the finite-time local convergence of a few AC variants with linear function approximation.

Under the two-timescale AC setting, [10] established the finite-time local convergence to a stationary point at a sample complexity of  $\tilde{\mathcal{O}}(\epsilon^{-2.5})$  with finite action space. [11] studied both local convergence and global convergence for two-timescale (natural) AC, with  $\tilde{\mathcal{O}}(\epsilon^{-2.5})$  and  $\tilde{\mathcal{O}}(\epsilon^{-4})$  sample complexity, respectively, under the discounted accumulated

reward. The algorithm collects multiple samples to update the critic. [9] established the global convergence of twotimescale AC methods for solving LQR, where they use a single sample to update the critic.

Under the single-timescale setting, [20] considered the least-squares temporal difference (LSTD) update for critic and obtained the optimal policy within the energy-based policy class for both linear function approximation and nonlinear function approximation using neural networks. In addition to the special implementation, [14] and [13] considered the single-timescale AC in general case, which have been clearly reviewed and compared in 1.1.

**Policy gradient methods.** The asymptotic convergence of policy gradient methods have been well established in [21, 22, 23, 16] via stochastic approximation methods [24]. Some recent works have shown that PG methods can find the global optimum of some particular class of problems, such as LQR [25, 26], the performance function of which satisfies the gradient dominance property [27], and tabular case RL problem [28]. Under general function approximation setting, finite-time convergence of PG methods have been provided in [28, 29, 30, 31]. Specifically, [28] established the finite-time convergence of PG methods under both tabular policy parameterizations and general parametric policy classes. [29] obtained an  $\epsilon$ -accurate stationary point for PG methods with a sample complexity of  $\mathcal{O}(\epsilon^{-2})$ , where they adopted Monte-Carlo sampling to find an unbiased estimation of policy gradient. Later, [30, 31] studied the variance reduction PG and acceleration PG.

**Notation.** Without other specification, for two sequences  $\{x_n\}$  and  $\{y_n\}$ , we write  $x_n = \mathcal{O}(y_n)$  if there exists an constant C such that  $x_n \leq Cy_n$ . We use  $\tilde{\mathcal{O}}(\cdot)$  to further hide logarithm factors. We use  $d_{TV}(\mu, v)$  to denote the total variation distance of two probability measure  $\mu$  and v, which is defined as  $d_{TV}(\mu, v) := \frac{1}{2} \int_{\mathcal{X}} |\mu(dx) - v(dx)|$ .

# 2 Preliminaries

In this section, we provide the background for single-timescale AC method.

#### 2.1 Markov decision process

We consider the reinforcement learning for the standard Markov Decision Process (MDP) defined by (S, A, P, r), where S is the state space and A is the action space. In this paper, we consider the finite action space  $|A| < \infty$ , while the state space can be either a finite set or an (unbounded) real vector space  $S \in \mathbb{R}^n$ .  $\mathcal{P}(s_{t+1}|s_t, a_t)$  denotes the transition kernel that the agent transits to state  $s_{t+1}$  after taking action  $a_t$  at current state  $s_t$ . Function  $r : S \times A \rightarrow$  $[-U_r, U_r]$  generates the reward of the agent taking action a at state s. A policy  $\pi_{\theta}(a|s)$  parameterized by  $\theta$  is defined as a mapping from a given state to a probability distribution over actions.

The RL problem of consideration aims to find a policy  $\pi_{\theta}$  that maximizes the infinite-horizon time-average reward [22, 12, 4, 10], which is given by

$$J(\theta) := \lim_{T \to \infty} \mathbb{E}_{\theta} \frac{\sum_{t=0}^{T-1} r(s_t, a_t)}{T} = \mathbb{E}_{s \sim \mu_{\theta}, a \sim \pi_{\theta}}[r(s, a)],$$
(1)

where  $\mu_{\theta}$  denotes the stationary state distribution induced by policy  $\pi_{\theta}$ , and the expectation  $\mathbb{E}_{\theta}$  is over the Markov chain under  $\pi_{\theta}$ . Hereafter, we refer to  $J(\theta)$  as the time-average reward (or exchangeably, performance function), which can be evaluated by the expected reward over the stationary distribution  $\mu_{\theta}$  and the policy  $\pi_{\theta}$  (the second equality in (1)). The existence of the stationary distribution can be guaranteed by the uniform ergodicity of the underlying MDP, which is a common assumption.

The state-value function is used to evaluate the overall rewards starting from state s and following policy  $\pi_{\theta}$  thereafter, which can be defined as

$$V_{\theta}(s) := \mathbb{E}_{\theta}\left[\sum_{t=0}^{\infty} (r(s_t, a_t) - J(\theta)) | s_0 = s\right],$$

where the action follows the policy  $a_t \sim \pi_{\theta}(\cdot|s_t)$  and the next state comes from the transition kernel  $s_{t+1} \sim \mathcal{P}(\cdot|s_t, a_t)$ . Similarly, we define the action-value (Q-value) function to evaluate the overall rewards starting from s, taking action a, and following policy  $\pi_{\theta}$  thereafter:

$$Q_{\theta}(s,a) = \mathbb{E}_{\theta}[\sum_{t=0}^{\infty} (r(s_t, a_t) - J(\theta)) | s_0 = s, a_0 = a]$$
  
=  $r(s,a) - J(\theta) + \mathbb{E}[V_{\theta}(s')],$  (2)

where the expectation is taken over  $s' \sim \mathcal{P}(\cdot|s, a)$ .

#### 2.2 Policy gradient theorem

A significant breakthrough in policy gradient methods is the policy gradient theorem [22], which provides an analytic expression for the gradient of performance function  $J(\theta)$  with respect to policy parameter  $\theta$ . Based on the above definitions, the policy gradient theorem takes the following form:

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{s \sim \mu_{\theta}, a \sim \pi_{\theta}} [Q_{\theta}(x, u) \nabla_{\theta} \log \pi_{\theta}(u|x)].$$
(3)

Optimizing  $J(\theta)$  with the gradient in (3) requires evaluating the Q-value of the current policy  $\pi_{\theta}$ , which is usually unknown. A natural idea is to use all the rewards collected along the sample trajectory (that is, the return) as an approximation to the true Q-value. This Monte Carlo-based episodic algorithm is known as the REINFORCE [21].

Note that for any function  $b : S \to \mathbb{R}$  independent of action a, we have

$$\sum_{a \in \mathcal{A}} b(s) \nabla \pi_{\theta}(a|s) = b(s) \nabla (\sum_{a \in \mathcal{A}} \pi_{\theta}(a|s)) = b(s) \nabla 1 = 0.$$

Therefore, the policy gradient theorem can be naturally generalized to add a comparison term b(s):

$$\nabla J(\theta) = \mathbb{E}_{s \sim \mu_{\theta}, a \sim \pi_{\theta}} [(Q_{\theta}(s, a) - b(s)) \nabla_{\theta} \log \pi_{\theta}(s|a)],$$

where b(s) is called the *baseline* function. A popular choice of *baseline* is the state-value function, which leads to the following advantage-based policy gradient

$$\nabla_{\theta} J(\theta) = \mathbb{E}_{s \sim \mu_{\theta}, a \sim \pi_{\theta}} [A_{\theta}(s, a) \nabla_{\theta} \log \pi_{\theta}(a|s)],$$

where  $A_{\theta} = Q_{\theta}(s, a) - V_{\theta}(s)$  is the advantage function.

This gives rise to the algorithm named "REINFORCE with baseline" [21]. In general, by introducing a baseline, the expected value of the actor update remains the same but the variance of the update can be reduced. However, like all Monte Carlo-based methods, it can still suffers from high variance and thus learns slowly. In addition, it is inconvenient to implement the algorithm online for continuing tasks [12].

Alternatively, AC methods add a parallel critic update to bootstrap the Q-value. We describe the classic singletimescale AC in the next subsection.

#### 2.3 The single-timescale actor-critic algorithm

We consider the practical single-sample single-timescale AC method, where the critic is bootstrap estimated using a single sample reward at each step, directly accommodating online learning for continuing tasks. We use the state-value function as a baseline, which is approximated by the following linear function:

$$\widehat{V}_{\theta}(s;\omega) = \phi(s)^{\top}\omega$$

To drive  $\widehat{V}_{\theta}(s; w)$  towards its true value V(s), the semi-gradient TD(0) update is applied to estimate the linear coefficient  $\omega$  (hereafter referred to as the critic):

$$\omega_{t+1} = \omega_t + \beta_t [(r_t - J(\theta) + \phi(s_{t+1})^\top \omega_t - \phi(s_t)^\top \omega_t)]\phi(s_t) = \omega_t + \beta_t [(r_t - J(\theta))\phi(s_t) + \phi(s_t)(\phi(s_{t+1}) - \phi(s_t))^\top)\omega_t],$$
(4)

where  $\beta_t$  is the step size of the critic  $\omega$  and  $r_t := r(s_t, a_t)$ . Since the time-average reward  $J(\theta)$  is unknown, an estimator  $\eta$  is introduced to estimate it. Hereafter, we refer to  $\eta$  as the time-average reward estimator, which is abbreviated to reward estimator. Therefore, the update rule can be written as

$$\omega_{t+1} = \omega_t + \beta_t [(r_t - \eta_t)\phi(s_t) + \phi(s_t)(\phi(s_{t+1}) - \phi(s_t))^\top)\omega_t], \eta_{t+1} = \eta_t + \gamma_t (r_t - \eta_t),$$

where  $\gamma_t$  is the step size of the reward estimator  $\eta_t$ .

Similar to REINFORCE with baseline, we define  $\delta_t := r_t - \eta_t + \phi(s_{t+1})^\top \omega_t - \phi(s_t)^\top \omega_t$  as an approximation to the advantage function and derive the corresponding update rule for actor:

$$\theta_{t+1} = \theta_t + \alpha_t \delta_t \nabla_\theta \log \pi_{\theta_t}(a_t | s_t),$$

where clearly  $\alpha_t$  is the actor stepsize. The above updates give rise to Algorithm 1, which is clearly introduced in [12] as a classic online one-step AC algorithm. Algorithm 1 can be efficiently implemented under both episodic and continuing setting due to its online nature.

#### Algorithm 1 Single-timescale Actor-Critic

- 1: Input initial actor parameter  $\theta_0$ , initial critic parameter  $\omega_0$ , initial reward estimator  $\eta_0$ , stepsize  $\alpha_t$  for actor,  $\beta_t$ for critic and  $\gamma_t$  for reward estimator.
- 2: Draw  $s_0$  from some initial distribution

3: for  $t = 0, 1, 2, \cdots, T - 1$  do

- Take the action  $a_t \sim \pi_{\theta_t}(\cdot|s_t)$ 4: Observe next state  $s_{t+1} \sim \mathcal{P}(\cdot|s_t, a_t)$  and the reward  $r_t = r(s_t, a_t)$   $\delta_t = r_t - \eta_t + \phi(s_{t+1})^\top \omega_t - \phi(s_t)^\top \omega_t$   $\eta_{t+1} = \eta_t + \gamma_t (r_t - \eta_t)$ 5:
- 6:
- 7:
- 8:
- $\begin{aligned} \omega_{t+1} &= \Pi_{U_{\omega}}(\omega_t + \beta_t \delta_t \phi(s_t)) \\ \theta_{t+1} &= \theta_t + \alpha_t \delta_t \nabla_{\theta} \log \pi_{\theta_t}(a_t | s_t) \end{aligned}$ 9:
- 10: end for

Note that the "single-timescale" refers to the fact that the stepsizes for the critic and the actor updates are constantly proportional. In addition, this is a "single-sample" algorithm, since only one sample is needed for update in each iteration. These considered settings are more practical than those performing multiple sampling and adopting least square temporal difference (LSTD) update for critic [20]. In Line 8 of Algorithm 1, a projection ( $\Pi_{U_{ij}}$ ) is introduced to keep the critic norm-bounded by  $U_{\omega}$ , which is common in the literature [10, 4, 11, 14]. In our analysis, the projection is relaxed using its non-expansive property.

Note that [10] provided the finite-time analysis for Algorithm 1 under the two-timescale setting, where the ratio between the actor and critic stepsizes are diminishing. In this work, we take a step further to show that Algorithm 1 can converge even under the more practical yet challenging single-timescale setting, under the same conditions assumed in [10]. Beyond that, we also improve the sample complexity by orders.

#### 3 Main Theory

#### 3.1 Assumptions

To further simplify the expression, we denote by s' the subsequent state-action pair of s. By taking the expectation of  $\omega_{t+1}$  in (4) with respect to the stationary distribution, for any given  $\omega_t$ , we have

$$\mathbb{E}[\omega_{t+1}|\omega_t] = \omega_t + \beta_t (b_\theta + A_\theta \omega_t), \tag{5}$$

where

$$A_{\theta} := \mathbb{E}_{(s,a,s')}[\phi(s)(\phi(s') - \phi(s))^{\top})],$$
  

$$b_{\theta} := \mathbb{E}_{(s,a)}[(r(s,a) - J(\theta))\phi(s)],$$
(6)

and  $s \sim \mu_{\theta}(\cdot), a \sim \pi_{\theta}(\cdot|s), s' \sim \mathcal{P}(\cdot|s, a)$ . It can easily shown that [12] the TD limiting point  $\omega_{\theta}^*$  satisfies:

$$b_{\theta} + A_{\theta}\omega_{\theta}^* = 0.$$

We define the following uniform upper bound for the critic approximation error:

$$\epsilon_{\mathrm{app}} := \sup_{\theta} \sqrt{\mathbb{E}_{s \sim \mu_{\theta}} (\phi(s)^{\top} \omega_{\theta}^* - V_{\theta}(s))^2}.$$

This error captures the quality of linear function approximation for critic. It can be expected that the learning errors of Algorithm 1 depends on how well the linear function can approximate the true state-value function  $V_{\theta}$ . The error  $\epsilon_{\text{app}}$  is zero if  $V_{\theta}$  is a linear function for any  $\theta$ .

The following assumptions are standard in the literature of analyzing AC methods with linear function approximation [20, 8, 10, 14, 13].

**Assumption 3.1** (Exploration). For any  $\theta$ , the matrix  $A_{\theta}$  defined in (6) is negative definite and its maximum eigenvalue can be upper bounded by  $-\lambda$ .

Assumption 3.1 is commonly adopted in analysing TD learning with learning function approximation [32, 33, 10, 34, 14, 13]. Such an assumption is made to guarantee the problem is solvable. As shown in [13] for tabular case, Assumption 3.1 holds if the policy  $\pi_{\theta}$  can explore all state-action pairs, which assures the exploration of  $\pi_{\theta}$ . From this assumption, we can choose  $U_{\omega} = \frac{2U_r}{\lambda}$  so that all  $\omega^*$  lie within the projection radius  $U_{\omega}$  because  $||b|| \leq 2U_r$  and  $||A^{-1}|| \le \lambda^{-1}$ , which justifies the projection operator introduced in Line 8 of Algorithm 1.

**Assumption 3.2** (Uniform ergodicity). For any  $\theta$ , denote  $\mu_{\theta}(\cdot)$  as the stationary distribution induced by the policy  $\pi_{\theta}(\cdot|s)$  and the transition probability measure  $\mathcal{P}(\cdot|s, a)$ . For a Markov chain generated by the policy  $\pi_{\theta}$  and transition kernel  $\mathcal{P}$ , there exists m > 0 and  $\rho \in (0, 1)$  such that

$$d_{TV}(\mathbb{P}(s_{\tau} \in \cdot | s_0 = s), \mu_{\theta}(\cdot)) \le m\rho^{\tau}, \forall \tau \ge 0, \forall s \in \mathcal{S}.$$

Assumption 3.2 assumes the Markov chain is geometrically mixing, which is commonly employed to characterize the noise induced by Markovian sampling. It is first introduced in [32] and widely used in the finite-time analysis of various RL algorithms with Markovian samples [33, 10, 14, 13].

**Assumption 3.3** (Lipschitz continuity of policy). Let  $\pi_{\theta}(a|s)$  be a policy parameterized by  $\theta \in \mathbb{R}^d$ . There exists positive constants  $B, L_l$  and  $L_{\pi}$  such that for all given state s and action a it holds that: i)  $\|\nabla \log \pi_{\theta}(a|s)\| \leq B, \forall \theta \in \mathbb{R}^d$ ; ii)  $\|\nabla \log \pi_{\theta}(a|s) - \nabla \log \pi_{\theta}(a|s)\| \leq L_l \|\theta_1 - \theta_2\|, \forall \theta_1, \theta_2 \in \mathbb{R}^d$ ; iii)  $\|\pi_{\theta_1}(a|s) - \pi_{\theta_2}(a|s)\| \leq L_{\pi} \|\theta_1 - \theta_2\|, \forall \theta_1, \theta_2 \in \mathbb{R}^d$ .

Assumption 3.3 is standard in the literature of policy gradient methods [35, 33, 29, 31, 10, 14, 13]. This assumption holds for many policies classes such as Gaussian policy [36], Boltzmann policy [15], and tabular softmax policy [28].

We end this section by emphasizing that our work requires only a subset of the assumptions made in the existing works on analyzing single-timescale AC [14, 13]. In particular, we do not require any of the strong assumptions on the stationary distribution that are made in [14, Assumption 11]. Compared with [13], we consider the more general continuous state-space beyond the restrictive tabular setting and consequently remove the non-redundancy assumption for the feature matrix (see [13, Assumption 5]). Compare to both works, we are able to analyze the more challenging Markovian sampling beyond the the i.i.d. sampling.

### 3.2 Main Theorem

To present our main result, we define an integer that depends on the number of total iteration T:

$$\tau_T := \min\{i \ge 0 | m \rho^{i-1} \le \frac{1}{\sqrt{T}}\},\$$

where  $m, \rho$  are constants defined in Assumption 3.2. Therefore, we choose  $\tau_T = \frac{\log m\rho^{-1}}{\log \rho^{-1}} + \frac{\log T}{2\log \rho^{-1}} = \mathcal{O}(\log T)$  such that  $m\rho^{\tau_T-1} \leq \frac{1}{\sqrt{T}}$ . The integer  $\tau_T$  represents the mixing time of an ergodic Markov chain, which will be used to control the Markovian noise in the analysis of the online AC algorithm.

We use the shorthand

$$y_t := \eta_t - J(\theta_t)$$

to denote the difference between the reward estimator at time t and the true time-average reward  $J(\theta_t)$ . We further use

$$z_t := \omega_t - \omega_t^*$$

with  $\omega_t^* := \omega_{\theta_t}^*$  to measure the error between the critic and its target value at iteration t.

**Theorem 3.4** (Markovian sampling). Suppose that all assumptions hold and choose  $\alpha_t = \frac{c_{\alpha}}{\sqrt{1+t}}, \beta_t = \gamma_t = \frac{1}{\sqrt{1+t}},$ where  $c_{\alpha}$  is a small positive constant. For Algorithm 1, when total iteration  $T \ge 2\tau_T$ , we have

$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$
$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2 = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$
$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2 = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}).$$

We defer the interpretation of the above results a bit to present the analysis results of the i.i.d. sampling first. For the i.i.d. sampling, the major difference from the Markovian sampling is that at the *t*-th iteration, the state  $s_t$  is sampled from the stationary distribution  $\mu_{\theta_t}$  instead of the evolving Markov chain (see Algorithm 2 in Appendix E). The i.i.d. sampling simplifies the analysis in which many noise terms reduce to zero effectively. This leads to an improved sample complexity compared to the Markovian sampling by up to logarithmic factors.

**Theorem 3.5** (i.i.d. sampling). Suppose that all assumptions hold and choose  $\alpha_t = \frac{c_{\alpha}}{\sqrt{1+t}}$ ,  $\beta_t = \gamma_t = \frac{1}{\sqrt{1+t}}$ , where  $c_{\alpha}$  is a small positive constant. For Algorithm 2, when total iteration  $T \ge 2\tau_T$ , we have

$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 = \mathcal{O}(\frac{1}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$
$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2 = \mathcal{O}(\frac{1}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$
$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2 = \mathcal{O}(\frac{1}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}).$$

The above results show that if the critic approximation error  $\epsilon_{app}$  is zero, the reward estimator, the critic, and the actor all converge at a sub-linear rate of  $\widetilde{\mathcal{O}}(T^{-\frac{1}{2}})$ . The additional logarithmic term hidden by  $\widetilde{\mathcal{O}}(\cdot)$  is the cost of the mixing time of the Markov chain, which can be removed under i.i.d. sampling. To put the results into perspective, note that  $\mathcal{O}(T^{-\frac{1}{2}})$  is the rate one would obtain from stochastic gradient descent (SGD) on a non-convex function with unbiased gradient updates. As a result, to obtain an  $\epsilon$ -approximate stationary point from Algorithm 1 and Algorithm 2, the corresponding sample complexity is  $\widetilde{\mathcal{O}}(\epsilon^{-2})$  for Markovian sampling and  $\mathcal{O}(\epsilon^{-2})$  for i.i.d. sampling, which matches the state-of-the-art performance of SGD on non-convex optimization problem.

This sample complexity compares favorably to other AC variants. Notably, [5] provided finite-time convergence for double-loop variant with a  $\mathcal{O}(\epsilon^{-4})$  sample complexity and [10] analysed two-timescale variant, yielding a  $\widetilde{\mathcal{O}}(\epsilon^{-2.5})$  sample complexity. The sample complexity gap is due to the inefficient usage of data. In double-loop setting, the critic starts over to estimate the Q-value for a fixed policy in the inner loop, ignoring the fact that the consecutive Q-values can be similar given relatively minor policy update. Besides, the two-timescale setting artificially slows down the actor by giving the actor a stepsize that decays slower than the critic, which in turn delays the learning. The single-timescale approach updates the critic and actor parallelly with proportional stepsizes and thus learns more efficiently.

It is worth mentioning that our result matches the  $O(\epsilon^{-2})$  sample complexity of policy gradient methods such as REINFORCE [7, 35]. It is previously found in [10] that there is a sample complexity gap between Algorithm 1 and REINFORCE [35], the former of which considered the two-timescale updates. In this paper, we fill this gap by giving an improved single-timescale analysis for Algorithm 1. We show that the practical AC methods can have the same sample complexity as REINFORCE.

#### 3.3 Proof Sketch

The main challenge in the finite-time analysis lies in that the estimation errors of the time-average reward, the critic, and the policy gradient are strongly coupled. To overcome this issue, we view the propagation of these errors as an interconnected system and analyze them comprehensively. To better appreciate the advantage of our analysis framework over the decoupled methods traditionally adopted in analyzing double-loop and two-timescale variants, we sketch the main proof steps of Theorem 3.4 in the following, where we also highlight the key challenges and techniques developed correspondingly. The supporting lemmas and theorems mentioned below can all be found in the Appendix.

We define three measures Y(T), Z(T), G(T) which denote the average values of the (time-average) reward estimation error, the critic error, and the square norm of the policy gradient, respectively:

$$Y(T) := \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}y_t^2, \ Z(T) := \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}\|z_t\|^2, \ G(T) := \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}\|\nabla J(\theta_t)\|^2.$$
(7)

We first derive implicit (coupled) upper bounds for the reward estimation error  $y_t$ , the critic error  $z_t$ , and the policy gradient  $\nabla J(\theta_t)$ , respectively. After that, we solve an interconnected system of inequalities in terms of Y(T), Z(T), G(T) to establish the finite-time convergence.

**Step 1: Reward estimation error analysis.** From the reward estimator update rule (Line 7 of Algorithm 1), we decompose the reward estimation error into:

$$y_{t+1}^2 = (1 - 2\gamma_t)y_t^2 + 2\gamma_t y_t (r_t - J(\theta_t)) + 2y_t (J(\theta_t) - J(\theta_{t+1})) + (J(\theta_t) - J(\theta_{t+1}) + \gamma_t (r_t - \eta_t))^2.$$
(8)

The second term on the right hand side of (8) is a bias term caused by the Markovian sample, which is characterized in Lemma C.1. As we shown in Lemma E.1, this bias reduces to 0 under i.i.d. sampling after taking the expectation.

The third term captures the variation of the moving targets  $J(\theta_t)$ . The double-loop approach runs a complete policy evaluation sub-problem in the inner loop for each target  $J(\theta_t)$  such that a relative accurate policy gradient easily ensures the monotonic decreasing of  $J(\theta_t)$ . The two-timescale approach requires  $\lim_{t\to\infty} \alpha_t/\beta_t = 0$  to guarantee this term converges to zero. In the case of single-timescale AC, we don't have the aforementioned specialized designs to facilitate the analysis. Instead, utilizing the smoothness of  $J(\theta)$ , we derive an implicit upper bound for this term as a function of the norm of  $y_t$  and  $\nabla J(\theta_t)$ . The last term in (8) reflects the variance in reward estimation, which is controlled by the diminishing stepsizes.

**Step 2: Critic error analysis.** By the critic update rule (Line 8 of Algorithm 1), we decompose the squared error by (neglecting the projection for the time being for the ease of comprehension)

$$\|z_{t+1}\|^2 = \|z_t\|^2 + 2\beta_t \langle z_t, \bar{g}(\omega_t, K_t) \rangle + 2\beta_t \Psi(O_t, \omega_t, K_t) + 2\beta_t \langle z_t, \Delta g(O_t, \eta_t, K_t) \rangle + 2\langle z_t, \omega_t^* - \omega_{t+1}^* \rangle + \|\beta_t(g(O_t, \omega_t, K_t) + \Delta g(O_t, \eta_t, K_t)) + (\omega_t^* - \omega_{t+1}^*)\|^2,$$
(9)

where  $O_t := (s_t, a_t, s_{t+1})$  is a tuple of observations and the definitions of  $g, \bar{g}, \Delta g$ , and  $\Psi$  can be found in (16) and (17) in Appendix A. Without diving into the detailed definitions, here we focus on illustrating the high-level insights of our proof. First of all, the second term on the right hand side of (9) can be bounded by  $-2\lambda\beta_t ||z_t||^2$  due to Assumption 3.1. It provides an explicit characterization of how sufficient exploration can help the convergence of learning. The third term is a Markovian noise, which is further bounded implicitly in Lemma C.3. For the i.i.d sampling case, as we show in Lemma E.1, this bias reduces to 0 after taking the expectation. The fourth term is caused by inaccurate reward and critic estimations, which can be bounded by the norm of  $y_t$  and  $z_t$ . The fifth term tracks both the critic estimation performance  $z_t$  and the difference between the drifting critic targets  $\omega_t^*$ . Similar to the case of Step 1, the double-loop approach bound this term relying on the accurate policy evaluation sub-problem in the inner loop for each target  $\omega_t^*$ , whereas the two-timescale approach ensures the convergence by additionally requiring  $\lim_{t\to\infty} \alpha_t/\beta_t = 0$ . In contrast, we establish an implicit bound for it by utilizing the Lipschitz continuity of the critic target provided in Lemma B.3. The last term reflects the variances of various estimations, which is bounded by the diminishing stepsizes.

**Step 3: Policy gradient norm analysis.** From the actor update rule (Line 9 of Algorithm 1) and the smoothness property of the performance function, we derive

$$|\nabla J(\theta_t)||^2 \leq \frac{1}{\alpha_t} (J(\theta_{t+1}) - J(\theta_t)) - \langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle + \Theta(O_t, \theta_t) - \langle \nabla J(\theta_t), \mathbb{E}_{O'_t} [\Delta h'(O'_t, \theta_t)] \rangle + \frac{L_{J'}}{2} \alpha_t \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2,$$

$$(10)$$

where  $O'_t$  is a shorthand for an independent sample from stationary distribution  $s \sim \mu_{\theta_t}$ ,  $a \sim \pi_{\theta_t}$ ,  $s' \sim \mathcal{P}$ ,  $\Theta$  is defined in (17), and  $L_{J'}$  is a constant. The first term on the right hand side of (10) compares the actor's performances between consecutive updates, which can be bounded via Abel summation by parts. The second term is an error introduced by the inaccurate estimations of both the time-average reward and the critic. This term was directly bounded to zero under both double-loop setting and two-timescale setting due to their particular algorithm design, to facilitate a decoupled analysis. We control this term by providing an implicit bound depending on  $y_t$ ,  $z_t$ , and  $\nabla J(\theta_t)$ . The third term is a noise term induced by Markovian sampling, which is characterized in Lemma C.5. Again, as proven in Lemma E.1, this bias reduces to 0 under i.i.d. sampling after taking the expectation. The fourth term comes from the linear function approximation error. The last term can be considered as the variance of the stochastic gradient update, which is controlled by the diminishing stepsizes.

Step 4: Interconnected iteration system analysis. Taking the expectation and summing (8), (9), and (10) from  $\tau_T$  to T-1, respectively, we obtain the following interconnected iteration system in terms of Y(T), Z(T), G(T):

$$Y(T) \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + a\sqrt{Y(T)G(T)},\tag{11}$$

$$Z(T) \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + bZ(T) + c\sqrt{Y(T)Z(T)},\tag{12}$$

$$G(T) \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) + d\sqrt{G(T)(2Y(T) + 8Z(T))},\tag{13}$$

where a, b, c, d are positive constants. By solving the above system of inequalities, we further prove that if 1 - 2b > 0and  $\frac{a}{2} + ad^2 + \frac{4ac^2d^2}{1-2b} < 1$ , then Y(T), Z(T), G(T) converge at a rate of  $\mathcal{O}(\frac{\log^2 T}{\sqrt{T}})$ . This condition can be easily satisfied by choosing the stepsize ratio  $c_{\alpha}$  to be smaller than a threshold given in (26). Thus, it completes the proof.

The above proof also applies to the i.i.d sampling case straightforwardly, with the corresponding terms pointed out in the above steps reducing to 0 in the analysis. The additional proof can be found in Lemma E.1.

# 4 Conclusion and Discussion

In this paper, we establish the first finite-time analysis for single-timescale AC method with Markovian sampling. Our work outperforms all the existing works in terms of performing online learning and requiring weaker assumptions. We provide a novel analysis framework that evaluates and controls the error propagation between time-average reward, actor, and critic, and establishes their convergence simultaneously. Our framework is general and may provide new insights for finite-time analysis of other single-timescale stochastic approximation algorithms.

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# **A** Notation

We make use of the following auxiliary Markov chain to deal with the Markovian noise.

### **Auxiliary Markov Chain:**

$$s_{t-\tau} \xrightarrow{\theta_{t-\tau}} a_{t-\tau} \xrightarrow{\mathcal{P}} s_{t-\tau+1} \xrightarrow{\theta_{t-\tau}} \widetilde{a}_{t-\tau+1} \xrightarrow{\mathcal{P}} \widetilde{s}_{t-\tau+2} \xrightarrow{\theta_{t-\tau}} \widetilde{a}_{t-\tau+2} \cdots \xrightarrow{\mathcal{P}} \widetilde{s}_t \xrightarrow{\theta_{t-\tau}} \widetilde{a}_t \xrightarrow{\mathcal{P}} \widetilde{s}_{t+1}.$$
(14)

For reference, we also show the original Markov chain.

#### **Original Markov Chain:**

$$s_{t-\tau} \xrightarrow{\theta_{t-\tau}} a_{t-\tau} \xrightarrow{\mathcal{P}} s_{t-\tau+1} \xrightarrow{\theta_{t-\tau+1}} \widetilde{a}_{t-\tau+1} \xrightarrow{\mathcal{P}} \widetilde{s}_{t-\tau+2} \xrightarrow{\theta_{t-\tau+2}} \widetilde{a}_{t-\tau+2} \cdots \xrightarrow{\mathcal{P}} \widetilde{s}_t \xrightarrow{\theta_t} \widetilde{a}_t \xrightarrow{\mathcal{P}} \widetilde{s}_{t+1}.$$
(15)

In the sequel, we denote by  $\tilde{O}_t := (\tilde{s}_t, \tilde{a}_t, \tilde{s}_{t+1})$  the tuple generated from the auxiliary Markov chain in (14) while  $O_t := (s_t, a_t, s_{t+1})$  denotes the tuple generated from the original Markov chain in (15).

We define the following functions, which will benefit to decompose the errors and simplify the presentation.

$$\Delta g(O, \eta, \theta) := [J(\theta) - \eta]\phi(s),$$

$$g(O, \omega, \theta) := [r(s, a) - J(\theta) + (\phi(s') - \phi(s))^{\top}\omega]\phi(s),$$

$$\bar{g}(\omega, \theta) := \mathbb{E}_{(s, a, s') \sim (\mu_{\theta}, \pi_{\theta}, \mathcal{P})}[[r(s, a) - J(\theta) + (\phi(s') - \phi(s))^{\top}\omega]\phi(s)],$$

$$\Delta h(O, \eta, \omega, \theta) := (J(\theta) - \eta + (\phi(s') - \phi(s))^{\top}(\omega - \omega^{*}(\theta))\nabla\log\pi_{\theta}(a|s),$$

$$\Delta h'(O, \theta) := ((\phi(s')\omega^{*}(\theta) - V_{\theta}(s')) - (\phi(s)^{\top}\omega^{*}(\theta) - V_{\theta}(s)))\nabla\log\pi_{\theta}(a|s),$$

$$h(O, \theta) := (r(s, a) - J(\theta) + \phi(s')^{\top}\omega^{*}(\theta) - \phi(s)^{\top}\omega^{*}(\theta))\nabla\log\pi_{\theta}(a|s).$$
(16)

We also define the following functions, which characterize the Markovian noise.

$$\Phi(O, \eta, \theta) := (\eta - J(\theta))(r(s, a) - J(\theta)), 
\Psi(O, \omega, \theta) := \langle \omega - \omega_{\theta}^{*}, g(O, \omega, \theta) - \bar{g}(\omega, \theta) \rangle, 
\Theta(O, O', \theta) := \langle \nabla J(\theta), \mathbb{E}_{O'}[h(O', \theta)] - h(O, \theta) \rangle,$$
(17)

where  $O'_t$  is a shorthand for an independent sample from stationary distribution  $s \sim \mu_{\theta_t}, a \sim \pi_{\theta_t}, s' \sim \mathcal{P}$ . Define  $U_{\delta} := 2U_r + 2U_{\omega}$  so that we have  $|\delta_t| \leq U_{\delta}$ , where  $\delta_t$  comes from Line 6 in Algorithm 1. Note that from Assumption 3.3, we have  $\|\delta \nabla \log \pi_{\theta}\| \leq G := U_{\delta}B$ .

# **B** Preliminary Lemmas

**Lemma B.1** ([10], Lemma C.4). For any  $\theta_1, \theta_2$ , we have

$$|J(\theta_1) - J(\theta_2)| \le L_J \|\theta_1 - \theta_2\|,$$

where  $L_J = 2U_r |\mathcal{A}| L_{\pi} (1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}).$ 

**Lemma B.2** ([29], Lemma 3.2). For the performance function  $J(\theta)$ , there exists a constant  $L_{J'} > 0$  such that for all  $\theta_1, \theta_2 \in \mathbb{R}^d$ , it holds that

$$\|\nabla J(\theta_1) - \nabla J(\theta_2)\| \le L_{J'} \|\theta_1 - \theta_2\|,\tag{18}$$

which further implies

$$J(\theta_2) \ge J(\theta_1) + \langle \nabla J(\theta_1), \theta_2 - \theta_1 \rangle - \frac{L_{J'}}{2} \|\theta_1 - \theta_2\|^2,$$
(19)

$$J(\theta_2) \le J(\theta_1) + \langle \nabla J(\theta_1), \theta_2 - \theta_1 \rangle + \frac{L_{J'}}{2} \|\theta_1 - \theta_2\|^2.$$

$$\tag{20}$$

**Lemma B.3** ([10], Proposition 4.4). *There exists a constant*  $L_* > 0$  *such that* 

$$\|\omega^*(\theta_1) - \omega^*(\theta_2)\| \le L_* \|\theta_1 - \theta_2\|, \forall \theta_1, \theta_2 \in \mathbb{R}^d,$$

where  $L_* = (2\lambda^{-2}U_r + 3\lambda^{-1}U_r)|\mathcal{A}|L_{\pi}(1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}).$ 

**Lemma B.4** ([33],[10]). For any  $\theta_1$  and  $\theta_2$ , it holds that

$$d_{TV}(\mu_{\theta_1},\mu_{\theta_2}) \leq |\mathcal{A}|(\lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) \|\theta_1 - \theta_2\|,$$
  
$$d_{TV}(\mu_{\theta_1} \otimes \pi_{\theta_1},\mu_{\theta_2} \otimes \pi_{\theta_2}) \leq |\mathcal{A}|L_{\pi}(1+\lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) \|\theta_1 - \theta_2\|,$$
  
$$d_{TV}(\mu_{\theta_1} \otimes \pi_{\theta_1} \otimes \mathcal{P},\mu_{\theta_2} \otimes \pi_{\theta_2} \otimes \mathcal{P}) \leq |\mathcal{A}|L_{\pi}(1+\lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) \|\theta_1 - \theta_2\|.$$

**Lemma B.5** ([10], Lemma B.2). Given time indexes t and  $\tau$  such that  $t \ge \tau > 0$ , consider the auxiliary Markov chain in (14). Conditioning on  $s_{t-\tau+1}$  and  $\theta_{t-\tau}$ , we have

$$\begin{aligned} d_{TV}(\mathbb{P}(s_{t+1} \in \cdot), \mathbb{P}(\widetilde{s}_{t+1} \in \cdot)) &\leq d_{TV}(\mathbb{P}(O_t \in \cdot), \mathbb{P}(O_t \in \cdot)), \\ d_{TV}(\mathbb{P}(O_t \in \cdot), \mathbb{P}(\widetilde{O}_t \in \cdot)) &= d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot), \mathbb{P}((\widetilde{s}_t, \widetilde{a}_t) \in \cdot)), \\ d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot), \mathbb{P}((\widetilde{s}_t, \widetilde{a}_t) \in \cdot)) &\leq d_{TV}(\mathbb{P}(s_t \in \cdot), \mathbb{P}(\widetilde{s}_t \in \cdot)) + \frac{1}{2} |\mathcal{A}|\mathbb{E}[\|\theta_t - \theta_{t-\tau}\|]. \end{aligned}$$

# C Proof of Main Theorem

#### C.1 Step 1: Reward estimation error analysis

In this subsection, we will establish an implicit bound for estimator.

**Lemma C.1.** From any  $t \ge \tau > 0$ , we have

$$\mathbb{E}[\Phi(O_t, \eta_t, \theta_t)] \le 4U_r L_J \|\theta_t - \theta_{t-\tau}\| + 2U_r |\eta_t - \eta_{t-\tau}| + 2U_r^2 |\mathcal{A}| L_\pi \sum_{i=t-\tau}^t \mathbb{E}\|\theta_i - \theta_{t-\tau}\| + 4U_r^2 m \rho^{\tau-1}.$$

**Theorem C.2.** Choose  $\alpha_t = \frac{c_{\alpha}}{\sqrt{t+1}}, \beta_t = \gamma_t = \frac{1}{\sqrt{t+1}}$ , we have

$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}y_t^2 \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + c_\alpha G(\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}\|\nabla J(\theta_t)\|^2)^{\frac{1}{2}}.$$
 (21)

Proof. From the update rule of reward estimator in Line 7 of Algorithm 1, we have

$$\eta_{t+1} - J(\theta_{t+1}) = \eta_t - J(\theta_t) + J(\theta_t) - J(\theta_{t+1}) + \gamma_t (r_t - \eta_t)$$

Then we have

$$y_{t+1}^2 = (y_t + J(\theta_t) - J(\theta_{t+1}) + \gamma_t (r_t - \eta_t))^2$$
  

$$\leq y_t^2 + 2y_t (J(\theta_t) - J(\theta_{t+1})) + 2\gamma_t y_t (r_t - \eta_t) + 2(J(\theta_t) - J(\theta_{t+1}))^2 + 2\gamma_t^2 (r_t - \eta_t)^2$$
  

$$= (1 - 2\gamma_t) y_t^2 + 2\gamma_t y_t (r_t - J(\theta_t)) + 2y_t (J(\theta_t) - J(\theta_{t+1})) + 2(J(\theta_t) - J(\theta_{t+1}))^2 + 2\gamma_t^2 (r_t - \eta_t)^2.$$

Taking expectation up to  $s_{t+1}$  (the whole trajectory), rearranging and summing from  $\tau_T$  to T-1, we have

$$\sum_{t=\tau_T}^{T-1} \mathbb{E}[y_t^2] \leq \underbrace{\sum_{t=\tau_T}^{T} \frac{1}{2\gamma_t} \mathbb{E}(y_t^2 - y_{t+1}^2)}_{I_1} + \underbrace{\sum_{t=\tau_T}^{T-1} \mathbb{E}[y_t(r_t - J(\theta_t))]}_{I_2} + \underbrace{\sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} \mathbb{E}[y_t(J(\theta_t) - J(\theta_{t+1}))]}_{I_3} + \underbrace{\sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} \mathbb{E}[(J(\theta_t) - J(\theta_{t+1}))^2]}_{I_4} + \underbrace{\sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} \mathbb{E}[(r_t - \eta_t)^2]}_{I_5}.$$

For term  $I_1$ , from Abel summation by parts, we have

$$I_{1} = \sum_{t=\tau_{T}}^{T-1} \frac{1}{2\gamma_{t}} (y_{t}^{2} - y_{t+1}^{2})$$
  
$$= \sum_{t=\tau_{T}+1}^{T-1} y_{t}^{2} (\frac{1}{2\gamma_{t}} - \frac{1}{2\gamma_{t-1}}) + \frac{1}{2\gamma_{\tau_{t}}} y_{\tau_{t}}^{2} - \frac{1}{\gamma_{T-1}} y_{T}^{2}$$
  
$$\leq \frac{2U_{r}^{2}}{\gamma_{T-1}}$$
  
$$= 2U_{r}^{2} \sqrt{T}.$$

For term  $I_2$ , from Lemma C.1, we have

$$\mathbb{E}[y_t(r_t - J(\theta_t))] \leq 4U_r L_J \|\theta_t - \theta_{t-\tau}\| + 2U_r |\eta_t - \eta_{t-\tau}| + 2U_r^2 |\mathcal{A}| L_\pi \sum_{i=t-\tau}^t \mathbb{E}\|\theta_i - \theta_{t-\tau}\| + 4U_r^2 m \rho^{\tau-1}$$
  
$$\leq 4U_r L_J G \tau \alpha_{t-\tau} + 4U_r^2 \tau \gamma_{t-\tau} + 2U_r^2 |\mathcal{A}| L_\pi \tau(\tau+1) G \alpha_{t-\tau} + 4U_r^2 m \rho^{\tau-1}$$
  
$$\leq (4U_r L_J G \tau + 2U_r^2 |\mathcal{A}| L_\pi G \tau(\tau+1)) \alpha_{t-\tau} + 4U_r^2 \tau \gamma_{t-\tau} + 4U_r^2 m \rho^{\tau-1}.$$

Choose  $\tau = \tau_T$ , we have

$$\begin{split} I_{2} &= \sum_{t=\tau_{T}}^{T-1} \mathbb{E}[y_{t}(r_{t} - J(\theta_{t}))] \\ &\leq (4U_{r}L_{J}G\tau_{T} + 2U_{r}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T} + 1)) \sum_{t=0}^{T-\tau_{T}-1} \alpha_{t} + 4U_{r}^{2}\tau_{T} \sum_{t=0}^{T-\tau_{T}-1} \gamma_{t} + 4U_{r}^{2} \sum_{t=\tau_{T}}^{T-1} \frac{1}{\sqrt{T}} \\ &\leq (8U_{r}L_{J}G\tau_{T} + 4U_{r}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T} + 1) + 8U_{r}^{2}\tau_{T} + 8U_{r}^{2})\sqrt{T-\tau_{T}}, \end{split}$$
 last inequality is due to

where the last inequality is due to

$$\sum_{t=0}^{T-\tau_T-1} \frac{1}{\sqrt{(1+t)}} \le \int_0^{T-\tau_T} t^{-\frac{1}{2}} dt \le 2\sqrt{T-\tau_T}.$$

For  $I_3$ , if  $y_t > 0$ , from (19), we have

$$y_t(J(\theta_t) - J(\theta_{t+1})) \le y_t(\frac{L_{J'}}{2} \|\theta_t - \theta_{t+1}\|^2 + \langle \nabla J(\theta_t), \theta_t - \theta_{t+1} \rangle) \\ \le L_{J'} U_r \|\theta_t - \theta_{t+1}\|^2 + |y_t| \|\theta_t - \theta_{t+1}\| \|\nabla J(\theta_t)\|.$$

If  $y_t \leq 0$ , from (20), we have

$$y_t(J(\theta_t) - J(\theta_{t+1})) \le y_t(-\frac{L_{J'}}{2} \|\theta_t - \theta_{t+1}\|^2 + \langle \nabla J(\theta_t), \theta_t - \theta_{t+1} \rangle) \\ \le L_{J'} U_r \|\theta_t - \theta_{t+1}\|^2 + |y_t| \|\theta_t - \theta_{t+1}\| \|\nabla J(\theta_t)\|.$$

Overall, we get

$$\begin{split} I_{3} &= \sum_{t=\tau_{T}}^{T-1} \frac{1}{\gamma_{t}} \mathbb{E}[y_{t}(J(\theta_{t}) - J(\theta_{t+1}))] \\ &\leq \sum_{t=\tau_{T}}^{T-1} \frac{1}{\gamma_{t}} \mathbb{E}[L_{J'}U_{r} \| \theta_{t} - \theta_{t+1} \|^{2} + |y_{t}| \| \theta_{t} - \theta_{t+1} \| \| \nabla J(\theta_{t}) \|] \\ &\leq \sum_{t=\tau_{T}}^{T-1} \mathbb{E}[c_{\alpha}L_{J'}U_{r}G^{2}\alpha_{t} + c_{\alpha}G|y_{t}| \| \nabla J(\theta_{t}) \|] \\ &\leq 2c_{\alpha}^{2}L_{J'}U_{r}G^{2}\sqrt{T-\tau_{T}} + c_{\alpha}G(\sum_{t=\tau_{T}}^{T-1} \mathbb{E}y_{t}^{2})^{\frac{1}{2}}(\sum_{t=\tau_{T}}^{T-1} \mathbb{E}\| \nabla J(\theta_{t}) \|^{2})^{\frac{1}{2}}. \end{split}$$

For term  $I_4$ , we have

$$\begin{split} I_4 &= \sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} \mathbb{E}[(J(\theta_t) - J(\theta_{t+1}))^2] \\ &\leq \sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} L_J^2 \mathbb{E} \| \theta_t - \theta_{t+1} \|^2 \\ &\leq \sum_{t=\tau_T}^{T-1} \frac{1}{\gamma_t} L_J^2 G^2 \alpha_t^2 \\ &= L_J^2 G^2 c_\alpha \sum_{t=\tau_T}^{T-1} \alpha_t \\ &\leq 2 L_J^2 G^2 c_\alpha^2 \sqrt{T - \tau_T}. \end{split}$$

For term  $I_5$ , we have

$$I_{5} = \sum_{t=\tau_{T}}^{T-1} \gamma_{t} \mathbb{E}[(r_{t} - J(\theta_{t}))^{2}]$$
$$\leq \sum_{t=\tau_{T}}^{T-1} 4U_{r}^{2} \gamma_{t}$$
$$\leq 8U_{r}^{2} \sqrt{T - \tau_{T}}.$$

Therefore, we get

$$\begin{split} \sum_{t=\tau_T}^{T-1} \mathbb{E}[y_t^2] &\leq (8U_r L_J G \tau_T + 4U_r^2 |\mathcal{A}| L_\pi G \tau_T (\tau_T + 1) + 8U_r^2 (\tau_T + 2) + 2c_\alpha^2 G^2 (L_{J'} U_r + L_J^2)) \sqrt{T - \tau_T} \\ &+ 2U_r^2 \sqrt{T} + c_\alpha G (\sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2)^{\frac{1}{2}}. \end{split}$$

Choose  $T \ge 2\tau_T$  such that  $\sqrt{T} \le 2\sqrt{T - \tau_T}$  and  $\frac{1}{\sqrt{T - \tau_T}} \le \frac{2}{\sqrt{T}}$ . Then we have

$$\begin{split} \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E}[y_t^2] &\leq (8U_r L_J G \tau_T + 4U_r^2 |\mathcal{A}| L_\pi G \tau_T (\tau_T + 1) + 8U_r^2 (\tau_T + 3) + 2c_\alpha^2 G^2 (L_{J'} U_r + L_J^2)) \frac{1}{\sqrt{T - \tau_T}} \\ &+ c_\alpha G (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} ||\nabla J(\theta_t)||^2)^{\frac{1}{2}} \\ &\leq 2(8U_r L_J G \tau_T + 4U_r^2 |\mathcal{A}| L_\pi G \tau_T (\tau_T + 1) + 8U_r^2 (\tau_T + 3) + 2c_\alpha^2 G^2 (L_{J'} U_r + L_J^2)) \frac{1}{\sqrt{T - \tau_T}} \\ &+ c_\alpha G (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} ||\nabla J(\theta_t)||^2)^{\frac{1}{2}} \\ &= \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + c_\alpha G (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} ||\nabla J(\theta_t)||^2)^{\frac{1}{2}}. \end{split}$$
Thus we finish the proof.

Thus we finish the proof.

## C.2 Step 2: Critic error analysis

In this subsection, we will establish an implicit upper bound for critic.

**Lemma C.3.** Given the definition of  $\Psi(\theta_t, \omega_t, O_t)$ , for any  $t \ge \tau > 0$ , we have

$$\mathbb{E}[\Psi(\theta_t,\omega_t,O_t)] \le C_1 \|\theta_t - \theta_{t-\tau}\| + U_{\delta}^2 |\mathcal{A}| L_{\pi} G\tau(\tau+1)\alpha_{t-\tau} + 2U_{\delta}^2 m \rho^{\tau-1} + 6U_{\delta} \|\omega_t - \omega_{t-\tau}\|,$$

where  $C_1 = 2U_{\delta}^2 |\mathcal{A}| L_{\pi} (1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) + 2U_{\delta}L_J + 2U_{\delta}L_*.$ **Theorem C.4.** Choose  $\alpha_t = \frac{c_{\alpha}}{\sqrt{t+1}}, \beta_t = \gamma_t = \frac{1}{\sqrt{t+1}}$ , we have

$$\frac{1}{T - \tau_T} \sum_{t=\tau_t}^t \mathbb{E} \|z_t\|^2 \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \frac{L_* G c_\alpha}{\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}} + \frac{1}{2\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}}.$$
(22)

## Proof. From the update rule of critic in Line 8 of Algorithm 1, we have

$$\begin{split} \|\omega_{t+1} - \omega_{t+1}^*\| &= \|\Pi_{U_{\omega}}(\omega_t + \beta_t \delta_t \phi(s_t)) - \omega_{t+1}^*\| \\ &= \|\Pi_{U_{\omega}}(\omega_t + \beta_t \delta_t \phi(s_t)) - \Pi_{U_{\omega}}(\omega_{t+1}^*)\| \\ &\leq \|\omega_t + \beta_t \delta_t \phi(s_t) - \omega_{t+1}^*\| \\ &= \|\omega_t + \beta_t (g(O_t, \omega_t, \theta_t) + \Delta g(O_t, \eta_t, \theta_t)) - \omega_{t+1}^*\| \\ &= \|\omega_t - \omega_t^* + \beta_t (g(O_t, \omega_t, \theta_t) + \Delta g(O_t, \eta_t, \theta_t)) + \omega_t^* - \omega_{t+1}^*\|. \end{split}$$

Therefore, we have

$$\begin{split} \|z_{t+1}\|^2 &= \|z_t + \beta_t (g(O_t, \omega_t, \theta_t) + \Delta g(O_t, \eta_t, \theta_t)) + \omega_t^* - \omega_{t+1}^* \|^2 \\ &= \|z_t\|^2 + 2\beta_t \langle z_t, g(O_t, \omega_t, \theta_t) \rangle + 2\beta_t \langle z_t, \Delta g(O_t, \eta_t, \theta_t) \rangle \\ &+ 2 \langle z_t, \omega_t^* - \omega_{t+1}^* \rangle + \|\beta_t (g(O_t, \omega_t, \theta_t) + \Delta g(O_t, \eta_t, \theta_t)) + \omega_t^* - \omega_{t+1}^* \|^2 \\ &= \|z_t\|^2 + 2\beta_t \langle z_t, \bar{g}(\omega_t, \theta_t) \rangle + 2\beta_t \Lambda (O_t, \omega_t, \theta_t) + 2\beta_t \langle z_t, \Delta g(O_t, \eta_t, \theta_t) \rangle \\ &+ 2 \langle z_t, \omega_t^* - \omega_{t+1}^* \rangle + \|\beta_t (g(O_t, \omega_t, \theta_t) + \Delta g(O_t, \eta_t, \theta_t)) + \omega_t^* - \omega_{t+1}^* \|^2 \\ &\leq \|z_t\|^2 + 2\beta_t \langle z_t, \bar{g}(\omega_t, \theta_t) \rangle + 2\beta_t \Lambda (O_t, \omega_t, \theta_t) + 2\beta_t \langle z_t, \Delta g(O_t, \eta_t, \theta_t) \rangle \\ &+ 2 \langle z_t, \omega_t^* - \omega_{t+1}^* \rangle + 2U_\delta^2 \beta_t^2 + 2\|\omega_t^* - \omega_{t+1}^* \|^2. \end{split}$$

Note that we have

$$\begin{aligned} \langle z_t, \bar{g}(\omega_t, \theta_t) \rangle &= \langle z_t, \bar{g}(\omega_t, \theta_t) - \bar{g}(\omega_t^*, \theta_t) \rangle \\ &= \langle z_t, \mathbb{E}[(\phi(s') - \phi(s))^\top (\omega_t - \omega_t^*)\phi(s)] \rangle \\ &= z_t^\top \mathbb{E}[\phi(s)(\phi(s') - \phi(s))^\top] z_t \\ &= z_t^\top A z_t \\ &\leq -\lambda \|z_t\|^2, \end{aligned}$$

where the first equation is due to the fact that  $\bar{g}(\omega_t^*, \theta_t) = 0$ . Taking expectation up to  $s_{t+1}$ , we have

$$\begin{split} \mathbb{E} \|z_{t+1}\|^{2} &\leq \mathbb{E} \|z_{t}\|^{2} + 2\beta_{t}\mathbb{E}\langle z_{t}, \bar{g}(\omega_{t}, \theta_{t})\rangle + 2\beta_{t}\mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t}) + 2\beta_{t}\mathbb{E}\langle z_{t}, \Delta g(O_{t}, \eta_{t}, \theta_{t})\rangle \\ &+ 2\mathbb{E}\langle z_{t}, \omega_{t}^{*} - \omega_{t+1}^{*}\rangle + 2U_{\delta}^{2}\beta_{t}^{2} + 2\mathbb{E} \|\omega_{t}^{*} - \omega_{t+1}^{*}\|^{2} \\ &\leq (1 - 2\lambda\beta_{t})\mathbb{E} \|z_{t}\|^{2} + 2\beta_{t}\mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t}) + 2\beta_{t}\mathbb{E}\langle z_{t}, \Delta g(O_{t}, \eta_{t}, \theta_{t})\rangle \\ &+ 2\mathbb{E}\langle z_{t}, \omega_{t}^{*} - \omega_{t+1}^{*}\rangle + 2U_{\delta}^{2}\beta_{t}^{2} + 2\mathbb{E} \|\omega_{t}^{*} - \omega_{t+1}^{*}\|^{2} \\ &\leq (1 - 2\lambda\beta_{t})\mathbb{E} \|z_{t}\|^{2} + 2\beta_{t}\mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t}) + 2\beta_{t}\mathbb{E} \|z_{t}\|\|y_{t}\| \\ &+ 2L_{*}\mathbb{E} \|z_{t}\| \cdot \|\theta_{t} - \theta_{t+1}\| + 2U_{\delta}^{2}\beta_{t}^{2} + 2L_{*}^{2}\mathbb{E} \|\theta_{t} - \theta_{t+1}\|^{2} \\ &\leq (1 - 2\lambda\beta_{t})\mathbb{E} \|z_{t}\|^{2} + 2\beta_{t}\mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t}) + 2\beta_{t}\mathbb{E} \|z_{t}\|\|y_{t}\| \\ &+ 2L_{*}G\alpha_{t}\mathbb{E} \|z_{t}\| + 2U_{\delta}^{2}\beta_{t}^{2} + 2L_{*}^{2}G^{2}\alpha_{t}^{2} \\ &\leq (1 - 2\lambda\beta_{t})\mathbb{E} \|z_{t}\|^{2} + 2\beta_{t}\mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t}) + 2\beta_{t}\sqrt{\mathbb{E}y_{t}^{2}}\sqrt{\mathbb{E}\|z_{t}\|^{2}} \\ &+ 2L_{*}G\alpha_{t}\sqrt{\mathbb{E}\|z_{t}\|^{2}} + 2U_{\delta}^{2}\beta_{t}^{2} + 2L_{*}^{2}G^{2}\alpha_{t}^{2}, \end{split}$$

where the last inequality comes from Cauchy-Schwarz inequality.

Rearranging and summing from  $\tau_T$  to T gives

$$2\lambda \sum_{\tau_T}^{T-1} \mathbb{E} \|z_t\|^2 \leq \underbrace{\sum_{t=\tau_T}^{T-1} \frac{1}{\beta_t} (\mathbb{E} \|z_t\|^2 - \mathbb{E} \|z_{t+1}\|^2)}_{I_1} + \underbrace{2\sum_{t=\tau_T}^{T-1} \mathbb{E} \Psi(O_t, \omega_t, \theta_t)}_{I_2} + \underbrace{\sum_{t=\tau_T}^{T-1} \sqrt{\mathbb{E} y_t^2} \sqrt{\mathbb{E} \|z_t\|^2}}_{I_3} + \underbrace{2L_* G c_\alpha \sum_{t=\tau_T}^{T-1} \sqrt{\mathbb{E} \|z_t\|^2}}_{I_4} + \underbrace{\sum_{t=\tau_T}^{T-1} (2U_\delta^2 \beta_t + 2L_*^2 G^2 c_\alpha \alpha_t)}_{I_5}.$$

In the sequel, we will control  $I_1, I_2, I_3, I_4, I_5$  respectively. For term  $I_1$ , from Abel summation by parts, we have

$$\begin{split} I_1 &= \sum_{t=\tau_T}^{T-1} \frac{1}{\beta_t} (\mathbb{E} \| z_t \|^2 - \mathbb{E} \| z_{t+1} \|^2) \\ &= \sum_{t=\tau_T+1}^{T-1} (\frac{1}{\beta_t} - \frac{1}{\beta_{t-1}}) \mathbb{E} \| z_t \|^2 + \frac{1}{\beta_{\tau_T}} \mathbb{E} \| z_{\tau_T} \|^2 - \frac{1}{\beta_{T-1}} \mathbb{E} \| z_T \|^2 \\ &\leq 4 U_{\omega}^2 (\sum_{t=\tau_T+1}^{T-1} (\frac{1}{\beta_t} - \frac{1}{\beta_{t-1}}) + \frac{1}{\beta_{\tau_T}}) \\ &= 4 U_{\omega}^2 \sqrt{T}, \end{split}$$

where the inequality is due to  $\mathbb{E}\|z_t\|^2 \leq 4U_{\omega}^2$  and discard the last term.

# For term $I_2$ , from Lemma C.3, choose $\tau = \tau_T$ , we have

$$\begin{split} \mathbb{E}\Psi(O_{t},\omega_{t},\theta_{t}) &\leq C_{1}\|\theta_{t}-\theta_{t-\tau_{T}}\| + U_{\delta}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T}+1)\alpha_{t-\tau_{T}} + 2U_{\delta}^{2}m\rho^{\tau_{T}-1} + 6U_{\delta}\|\omega_{t}-\omega_{t-\tau_{T}}\|\\ &\leq C_{1}\sum_{k=t-\tau_{T}}^{t-1}G\alpha_{k} + U_{\delta}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T}+1)\alpha_{t-\tau_{T}} + \frac{2U_{\delta}^{2}}{\sqrt{T}} + 6U_{\delta}\sum_{k=t-\tau_{T}}^{t-1}U_{\delta}\beta_{k}\\ &\leq (C_{1}G\tau_{T}+U_{\delta}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T}+1))\alpha_{t-\tau_{T}} + \frac{2U_{\delta}^{2}}{\sqrt{T}} + 6U_{\delta}^{2}\tau_{T}\beta_{t-\tau_{T}}. \end{split}$$

Then we get

$$I_{2} = 2 \sum_{T=\tau_{T}}^{T-1} \mathbb{E}\Psi(O_{t}, \omega_{t}, \theta_{t})$$
  
$$\leq 2 \sum_{T=\tau_{T}}^{T-1} (C_{1}G\tau_{T} + U_{\delta}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T}+1))\alpha_{t-\tau} + \frac{2U_{\delta}^{2}}{\sqrt{T}} + 6U_{\delta}^{2}\tau_{T}\beta_{t-\tau_{T}}$$
  
$$\leq (4C_{1}G\tau_{T}c_{\alpha} + 4U_{\delta}^{2}|\mathcal{A}|L_{\pi}G\tau_{T}(\tau_{T}+1)c_{\alpha} + 2U_{\delta}^{2} + 12U_{\delta}^{2}\tau_{T})\sqrt{T-\tau_{T}}.$$

For term  $I_3$ , we have

$$I_{3} = \sum_{t=\tau_{T}}^{T-1} \sqrt{\mathbb{E}y_{t}^{2}} \sqrt{\mathbb{E}\|z_{t}\|^{2}}$$
$$\leq (\sum_{t=\tau_{T}}^{T-1} \mathbb{E}y_{t}^{2})^{\frac{1}{2}} (\sum_{t=\tau_{T}}^{T-1} \mathbb{E}\|z_{t}\|^{2})^{\frac{1}{2}}.$$

For term  $I_4$ , we have

$$I_{4} = 2L_{*}Gc_{\alpha} \sum_{t=\tau_{T}}^{T-1} \sqrt{\mathbb{E} \|z_{t}\|^{2}}$$
  
$$\leq 2L_{*}Gc_{\alpha} (\sum_{t=\tau_{T}}^{T-1} 1)^{\frac{1}{2}} (\sum_{t=\tau_{T}}^{T-1} \mathbb{E} \|z_{t}\|^{2})^{\frac{1}{2}}$$
  
$$= 2L_{*}Gc_{\alpha} \sqrt{T-\tau_{T}} (\sum_{t=\tau_{T}}^{T-1} \mathbb{E} \|z_{t}\|^{2})^{\frac{1}{2}}.$$

For term  $I_5$ , we have

$$I_{5} = \sum_{t=\tau_{T}}^{T-1} (2U_{\delta}^{2}\beta_{t} + 2L_{*}^{2}G^{2}c_{\alpha}\alpha_{t})$$
$$\leq (4U_{\delta}^{2} + 4L_{*}^{2}G^{2}c_{\alpha}^{2})\sqrt{T-\tau_{T}}.$$

Overall, we get

$$2\lambda \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2 \le 4U_{\omega}^2 \sqrt{T} + (4C_1 G\tau_T c_{\alpha} + 4U_{\delta}^2 |\mathcal{A}| L_{\pi} G\tau_T (\tau_T + 1) c_{\alpha} + 6U_{\delta}^2 + 12U_{\delta}^2 \tau_T + 4L_*^2 G^2 c_{\alpha}^2) \sqrt{T - \tau_T} + (\sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}} + 2L_* Gc_{\alpha} \sqrt{T - \tau_T} (\sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}}.$$

Therefore, we have

$$\begin{aligned} \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2 &\leq \frac{1}{\lambda} (4U_{\omega}^2 + 4C_1 G \tau_T c_{\alpha} + 4U_{\delta}^2 |\mathcal{A}| L_{\pi} G \tau_T (\tau_T + 1) c_{\alpha} + 6U_{\delta}^2 + 12U_{\delta}^2 \tau_T + 4L_*^2 G^2 c_{\alpha}^2) \frac{1}{\sqrt{T}} \\ &+ \frac{L_* G c_{\alpha}}{\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}} + \frac{1}{2\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}} \\ &= \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \frac{L_* G c_{\alpha}}{\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}} \\ &+ \frac{1}{2\lambda} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2)^{\frac{1}{2}} (\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}}. \end{aligned}$$

## C.3 Step 3: Policy gradient norm analysis

In this subsection, we will establish an implicit upper bound for policy gradient norm. Lemma C.5. For any  $t \ge \tau > 0$ , it holds that

$$\mathbb{E}[\Theta(O_t, \theta_t)] \le D_1(\tau + 1) \sum_{k=t-\tau+1}^t \mathbb{E} \|\theta_k - \theta_{k-1}\| + D_2 m \rho^{\tau-1},$$

where  $D_1 = \max\{U_{\delta}BL_{J'} + 3L_JL_h, 2U_{\delta}BL_J|A|L_{\pi}\}$  and  $D_2 = 4U_{\delta}BL_J$ . **Theorem C.6.** We have

$$\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2 \leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) \\
+ B(\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2)^{\frac{1}{2}} (\frac{2}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 + \frac{8}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}}.$$
(23)

Proof. From the update rule of actor in Line 9 of Algorithm 1 and 19, we have

$$\begin{split} J(\theta_{t+1}) &\geq J(\theta_t) + \langle \nabla J(\theta_t), \theta_{t+1} - \theta_t \rangle - \frac{L_{J'}}{2} \|\theta_1 - \theta_2\|^2 \\ &= J(\theta_t) + \langle \nabla J(\theta_t), \delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \rangle - \frac{L_{J'}}{2} \alpha_t^2 \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2 \\ &= J(\theta_t) + \alpha_t \langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle + \alpha_t \langle \nabla J(\theta_t), h(O_t, \theta_t) \rangle - \frac{L_{J'}}{2} \alpha_t^2 \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2 \\ &= J(\theta_t) + \alpha_t \langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle - \alpha_t \Theta(O_t, \theta_t) \\ &+ \alpha_t \langle \nabla J(\theta_t), \mathbb{E}_{O'}[h(O', \theta_t)] \rangle - \frac{L_{J'}}{2} \alpha_t^2 \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2 \\ &= J(\theta_t) + \alpha_t \langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle - \alpha_t \Theta(O_t, \theta_t) + \alpha_t \|\nabla J(\theta_t)\|^2 \\ &+ \alpha_t \langle \nabla J(\theta_t), \mathbb{E}_{O'}[\Delta h'(O', \theta_t)] \rangle - \frac{L_{J'}}{2} \alpha_t^2 \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2, \end{split}$$

where the last equality is due to the fact

 $\mathbb{E}_{O'}[h(O',\theta) - \Delta h'(O',\theta)] = \mathbb{E}_{O'}[(r(s,a) - J(\theta) + V_{\theta}(s') - V_{\theta}(s))\nabla \log \pi_{\theta}(a|s)] = \nabla J(\theta).$ Rearranging the above inequality and taking expectation, we have

$$\mathbb{E} \|\nabla J(\theta_t)\|^2 \leq \frac{1}{\alpha_t} (\mathbb{E} [J(\theta_{t+1}) - J(\theta_t)]) - \mathbb{E} \langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle + \mathbb{E} [\Theta(O_t, \theta_t)] \\ - \mathbb{E} \langle \nabla J(\theta_t), \mathbb{E}_{O'} [\Delta h'(O', \theta_t)] \rangle + \frac{L_{J'}}{2} \alpha_t \mathbb{E} \|\delta_t \nabla \log \pi_{\theta_t}(a_t | s_t) \|^2.$$

Note that from Cauchy-Schwartz inequality, we have

$$-\mathbb{E}\langle \nabla J(\theta_t), \Delta h(O_t, \eta_t, \omega_t, \theta_t) \rangle \le B\sqrt{\mathbb{E}\|\nabla J(\theta_t)\|^2}\sqrt{2\mathbb{E}y_t^2 + 8\mathbb{E}\|z_t\|^2}.$$

From Lemma C.5 and choosing  $\tau = \tau_T$ , we have

$$\mathbb{E}[\Theta(O_t, \theta_t)] \le D_1(\tau_T + 1) \sum_{k=t-\tau_T+1}^t \mathbb{E} \|\theta_k - \theta_{k-1}\| + D_2 m \rho^{\tau_T - 1}$$
  
$$\le D_1(\tau_T + 1) G \sum_{k=t-\tau_T}^{t-1} \alpha_k + D_2 m \rho^{\tau_T - 1}$$
  
$$\le G D_1(\tau_T + 1)^2 \alpha_{t-\tau_T} + D_2 \frac{1}{\sqrt{T}}.$$

Furthermore, it holds that

$$\begin{split} \mathbb{E}_{O'} \|\nabla h'(O,\theta)\|^2 &= \mathbb{E}_{O'} \|((\phi(s')^\top \omega^* - V_{\theta}(s')) - (\phi(s)^\top \omega^* - V_{\theta}(s))) \nabla \log \pi_{\theta}(a|s)\|^2 \\ &\leq \mathbb{E}_{O'} [B^2((\phi(s')^\top \omega^* - V_{\theta}(s')) - (\phi(s)^\top \omega^* - V_{\theta}(s)))^2] \\ &\leq \mathbb{E}_{O'} [2B^2(\phi(s')^\top \omega^* - V_{\theta}(s'))^2 + (\phi(s)^\top \omega^* - V_{\theta}(s))^2] \\ &= 4B^2 \mathbb{E}_{O'} [(\phi(s)^\top \omega^*(\theta) - V_{\theta}(s))^2] \\ &= 4B^2 \epsilon_{app}^2. \end{split}$$

Therefore, we have

$$\begin{aligned} -\langle \nabla J(\theta_t), \mathbb{E}_{O'}[\Delta h'(O', \theta_t)] \rangle &\leq L_J \sqrt{\|\mathbb{E}_{O'}[\Delta h'(O_t, \theta_t)]\|^2} \\ &\leq L_J \sqrt{\mathbb{E}_{O'}\|\Delta h'(O_t, \theta_t)\|^2} \\ &\leq 2BL_J \epsilon_{\mathrm{app}}, \end{aligned}$$

where we use  $\|\nabla J(\theta)\| \leq L_J$  which comes from lemma B.1. Plugging the three terms yields

$$\mathbb{E} \|\nabla J(\theta_t)\|^2 \leq \frac{1}{\alpha_t} (\mathbb{E}[J(\theta_{t+1})] - \mathbb{E}[J(\theta_t)]) + B\sqrt{\mathbb{E} \|\nabla J(\theta_t)\|^2} \sqrt{2\mathbb{E}y_t^2 + 8\mathbb{E} \|z_t\|^2} + 2BL_J \epsilon_{\mathrm{app}} + GD_1(\tau_T + 1)^2 \alpha_{t-\tau_T} + D_2 \frac{1}{\sqrt{T}} + \frac{L_{J'}}{2} G^2 \alpha_t.$$

Summing over t from  $\tau_T$  to T-1 gives

$$\begin{split} \sum_{t=\tau_{T}}^{T-1} \mathbb{E} \|\nabla J(\theta_{t})\|^{2} &\leq \underbrace{\sum_{t=\tau_{T}}^{T-1} \frac{1}{\alpha_{t}} (\mathbb{E}[J(\theta_{t+1}) - \mathbb{E}[J(\theta_{t})])}_{I_{1}} + B \sum_{t=\tau_{T}}^{T-1} \sqrt{\mathbb{E} \|\nabla J(\theta_{t})\|^{2}} \sqrt{2\mathbb{E}y_{t}^{2} + 8\mathbb{E} \|z_{t}\|^{2}} \\ &+ \underbrace{\sum_{t=\tau_{T}}^{T-1} GD_{1}(\tau_{T}+1)^{2} \alpha_{t-\tau_{T}}}_{I_{2}} + D_{2} \sqrt{T - \tau_{T}} + 2BL_{J} \epsilon_{\mathrm{app}}(T - \tau_{T}). \end{split}$$

In the following, we will bound  $I_1, I_2$  respectively.

For term  $I_1$ , from Abel summation by parts, we have

$$\begin{split} I_1 &= \sum_{t=\tau_T}^{T-1} \frac{1}{\alpha_t} (\mathbb{E}[J(\theta_{t+1}) - \mathbb{E}[J(\theta_t)]) \\ &= \sum_{t=\tau_T+1}^{T-1} (\frac{1}{\alpha_{t-1}} - \frac{1}{\alpha_t}) \mathbb{E}[J(\theta_t)] - \mathbb{E}[J(\theta_{\tau_T})] \frac{1}{\alpha_{\tau_T}} + \frac{1}{\alpha_{T-1}} \mathbb{E}[J(\theta_T)] \\ &\leq \sum_{t=\tau_T+1}^{T-1} (\frac{1}{\alpha_t} - \frac{1}{\alpha_{t-1}}) U_r + \frac{1}{\alpha_{\tau_T}} U_r + \frac{1}{\alpha_{T-1}} U_r \\ &= \frac{2U_r}{\alpha_{T-1}} \\ &= \frac{2U_r}{c_\alpha} \sqrt{T}. \end{split}$$

For term  $I_2$ , we have

$$I_{2} = \sum_{t=\tau_{T}}^{T-1} GD_{1}(\tau_{T}+1)^{2} \alpha_{t-\tau_{T}}$$
$$\leq 2GD_{1}(\tau_{T}+1)^{2} \sqrt{T-\tau_{T}}.$$

Overall, we have

$$\begin{split} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2 &\leq \frac{2U_r}{c_{\alpha}} \sqrt{T} + (2GD_1(\tau_T+1)^2 + D_2) \sqrt{T - \tau_T} + 2BL_J \epsilon_{\mathrm{app}}(T - \tau_T) \\ &+ B \sum_{t=\tau_T}^{T-1} \sqrt{\mathbb{E} \|\nabla J(\theta_t)\|^2} \sqrt{2\mathbb{E} y_t^2 + 8\mathbb{E} \|z_t\|^2} \\ &\leq \frac{2U_r}{c_{\alpha}} \sqrt{T} + (2GD_1(\tau_T+1)^2 + D_2) \sqrt{T - \tau_T} + 2BL_J \epsilon_{\mathrm{app}}(T - \tau_T) \\ &+ B (\sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2)^{\frac{1}{2}} (2\sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 + 8\sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2)^{\frac{1}{2}}. \end{split}$$

Therefore, we get

$$\begin{aligned} \frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2 &\leq \left(\frac{4U_r}{c_\alpha} + 4GD_1(\tau_T + 1)^2 + 2D_2\right) \frac{1}{\sqrt{T}} + 2BL_J \epsilon_{\text{app}} \\ &+ B\left(\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2\right)^{\frac{1}{2}} \left(\frac{2}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 + \frac{8}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2\right)^{\frac{1}{2}} \\ &= \mathcal{O}\left(\frac{\log^2 T}{\sqrt{T}}\right) + \mathcal{O}(\epsilon_{\text{app}}) \\ &+ B\left(\frac{1}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|\nabla J(\theta_t)\|^2\right)^{\frac{1}{2}} \left(\frac{2}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} y_t^2 + \frac{8}{T - \tau_T} \sum_{t=\tau_T}^{T-1} \mathbb{E} \|z_t\|^2\right)^{\frac{1}{2}}, \end{aligned}$$

which concludes the proof.

# C.4 Step 4: Interconnected iteration system analysis

In this subsection, we perform an interconnected iteration system analysis to prove Theorem 3.4. **Proof of Theorem 3.4.** 

Proof. Combining (21), (22), and (23), we have

$$\begin{split} Y(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + c_{\alpha}G\sqrt{Y(T)G(T)}, \\ Z(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \frac{L_*Gc_{\alpha}}{\lambda}Z(T) + \frac{1}{2\lambda}\sqrt{Y(T)Z(T)}, \\ G(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) + B\sqrt{G(T)(2Y(T) + 8Z(T))}. \end{split}$$

Denote

$$a := c_{\alpha}G,$$
  

$$b := \frac{L_*Gc_{\alpha}}{\lambda},$$
  

$$c := \frac{1}{2\lambda},$$
  

$$d := B.$$

Then we have

$$\begin{split} Y(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + a\sqrt{Y(T)G(T)}, \\ Z(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + bZ(T) + c\sqrt{Y(T)Z(T)}, \\ G(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) + d\sqrt{G(T)(2Y(T) + 8Z(T))}. \end{split}$$

For G(T), we get

$$G(T) \leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) + \frac{1}{2}G(T) + d^2(Y(T) + 4Z(T)),$$
  

$$G(T) \leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}) + 2d^2(Y(T) + 4Z(T)).$$
(24)

For Z(T), we have

$$Z(T) \leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + bZ(T) + \frac{1}{2}Z(T) + \frac{c^2}{2}Y(T).$$

If 1 - 2b > 0, we further have

$$Z(T) \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \frac{c^2}{1-2b}Y(T).$$
(25)

For Y(T), we get

$$Y(T) \le \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \frac{a}{2}(Y(T) + G(T)).$$

Plugging (24) and (25) into the above inequality gives

$$\begin{split} Y(T) &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\rm app}) + \frac{a}{2}(Y(T) + 2d^2Y(T) + 8d^2Z(T)) \\ &\leq \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\rm app}) + \frac{a}{2}(Y(T) + 2d^2Y(T) + \frac{8d^2c^2}{1 - 2b}Y(T)) \\ &= \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\rm app}) + (\frac{a}{2} + ad^2 + \frac{4ac^2d^2}{1 - 2b})Y(T). \end{split}$$

Therefore, if  $\frac{a}{2} + ad^2 + \frac{4ac^2d^2}{1-2b} < 1,$  we have

$$Y(T) = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\mathrm{app}})$$

According to the definition of a, b, c, d, we have

$$\frac{a}{2} + ad^2 + \frac{4ac^2d^2}{1-2b} = c_\alpha G(\frac{1}{2} + B^2 + \frac{B^2}{\lambda^2 - 2\lambda L_*Gc_\alpha}).$$

Since we have to satisfy 1-2b > 0, thus we choose  $c_{\alpha}$  small enough such that  $1-2b \ge \frac{1}{2}$ , which implies

$$c_{\alpha} \leq \frac{\lambda}{4L_*G}$$

Therefore, we further have

$$\frac{a}{2} + ad^2 + \frac{4ac^2d^2}{1 - 2b} = c_{\alpha}G(\frac{1}{2} + B^2 + \frac{B^2}{\lambda^2 - 2\lambda L_*Gc_{\alpha}})$$
$$\leq c_{\alpha}G(\frac{1}{2} + B^2 + \frac{2B^2}{\lambda^2}).$$

To satisfy  $c_{\alpha}G(\frac{1}{2}+B^2+\frac{2B^2}{\lambda^2})<1$ , we choose

$$c_{\alpha} < \frac{2\lambda^2}{G(\lambda^2 + 2B^2\lambda^2 + 4B^2)}$$

Overall, we choose

$$c_{\alpha} < \min\{\frac{\lambda}{4L_*G}, \frac{2\lambda^2}{G(\lambda^2 + 2B^2\lambda^2 + 4B^2)}\}.$$
(26)

Therefore, we have

$$Y(T) = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$

and consequently,

$$Z(T) = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}),$$
$$G(T) = \mathcal{O}(\frac{\log^2 T}{\sqrt{T}}) + \mathcal{O}(\epsilon_{\text{app}}).$$

Thus we conclude our proof.

# **D Proof of Supporting Lemmas**

The following three lemmas only deal with the Markovian noise, which are originally proved in [10] and updated in [37]. We include the proof with slight modifications for proving Theorem 3.4.

# Proof of Lemma C.1.

*Proof.* We will divide the proof of this lemma into four steps.

**Step 1:** show that for any  $\theta_1, \theta_2, \eta, O = (s, a, s')$ , we have

$$|\Phi(O,\eta,\theta_1) - \Phi(O,\eta,\theta_2)| \le 4U_r L_J \|\theta_1 - \theta_2\|.$$
(27)

By the definition of  $\Phi(O, \eta, \theta)$  in (17), we have

$$\begin{split} \Phi(O,\eta,\theta_1) - \Phi(O,\theta,\theta_2) &|= |(\eta - J(\theta_1))(r - J(\theta_1)) - (\eta - J(\theta_2))(r - J(\theta_2))| \\ &\leq |(\eta - J(\theta_1))(r - J(\theta_1)) - (\eta - J(\theta_1))(r - J(\theta_2))| \\ &+ |(\eta - J(\theta_1))(r - J(\theta_2)) - (\eta - J(\theta_2))(r - J(\theta_2))| \\ &\leq 4U_r |J(\theta_1) - J(\theta_2)| \\ &\leq 4U_r L_J ||\theta_1 - \theta_2||. \end{split}$$

**Step 2:** show that for any  $\theta$ ,  $\eta_1$ ,  $\eta_2$ , O, we have

$$\Phi(O, \eta_1, \theta) - \Phi(O, \eta_2, \theta) \le 2U_r |\eta_1 - \eta_2|.$$
(28)

By definition, we have

$$\begin{split} |\Phi(O,\eta_1,\theta) - \Phi(O,\eta_2,\theta)| &= |(\eta_1 - J(\theta))(r - J(\theta)) - (\eta_2 - J(\theta))(r - J(\theta)) \\ &\leq 2U_r |\eta_1 - \eta_2|. \end{split}$$

**Step 3:** show that for original tuple  $O_t$  and the auxiliary tuple  $\tilde{O}_t$ , conditioned on  $s_{t-\tau-1}$  and  $\theta_{t-\tau}$ , we have

$$|\mathbb{E}[\Phi(O_t, \eta_{t-\tau}, \theta_{t-\tau}) - \mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau})]| \le 2U_r^2 |\mathcal{A}| L_\pi \sum_{k=t-\tau}^{\iota} \mathbb{E}\|\theta_k - \theta_{t-\tau}\|.$$
(29)

By definition, we have

 $\mathbb{E}[\Phi(O_t, \eta_{t-\tau}, \theta_{t-\tau}) - \mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau})] = (\eta_{t-\tau} - J(\theta_{t-\tau}))\mathbb{E}[r(s_t, a_t) - r(\widetilde{s}_t, \widetilde{a}_t)].$ By definition of total variation norm, we have

$$\mathbb{E}[r(s_t, a_t) - r(\widetilde{s}_t, \widetilde{a}_t)] \le 2U_r d_{TV}(\mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(\widetilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})).$$
(30)  
5 we get

By Lemma B.5, we get

$$\begin{aligned} &d_{TV}(\mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})) \\ &= d_{TV}(\mathbb{P}((s_t, a_t) \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}((\widetilde{s}_t, \widetilde{a}_t) \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})) \\ &\leq d_{TV}(\mathbb{P}(s_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(\widetilde{s}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})) + \frac{1}{2}L_{\pi}\mathbb{E}\|\theta_t - \theta_{t-\tau}\| \\ &\leq d_{TV}(\mathbb{P}(O_{t-1} \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(\widetilde{O}_{t-1} \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})) + \frac{1}{2}L_{\pi}\mathbb{E}\|\theta_t - \theta_{t-\tau}\|. \end{aligned}$$

Repeat the above argument from t to  $t - \tau + 1$ , we have

$$d_{TV}(\mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(\widetilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})) \le \frac{1}{2} |\mathcal{A}| \sum_{k=t-\tau}^t \mathbb{E} \|\theta_k - \theta_{t-\tau}\|.$$
(31)

Plugging (31) into (30), we have

$$|\mathbb{E}[\Phi(O_t, \eta_{t-\tau}, \theta_{t-\tau}) - \mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau})]| \le 2U_r^2 |\mathcal{A}| L_{\pi} \sum_{k=t-\tau}^t \mathbb{E}||\theta_k - \theta_{t-\tau}||$$

**Step 4:** show that conditioned on  $s_{t-\tau+1}$  and  $\theta_{t-\tau}$ , we have

$$\mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau})] \le 4U_r^2 m \rho^{\tau-1}.$$
(32)

Note that according to definition, we have

$$\mathbb{E}[\Phi(O'_{t-\tau}, \eta_{t-\tau}, \theta_{t-\tau})|\theta_{t-\tau}] = 0$$

where  $O'_{t-\tau} = (s'_{t-\tau}, a'_{t-\tau}, s'_{t-\tau+1})$  is the tuple generated by  $s'_{t-\tau} \sim \mu_{t-\tau}, a'_{t-\tau} \sim \pi_{\theta_{t-\tau}}, s'_{t-\tau+1} \sim \mathcal{P}$ . From the uniform ergodicity in Assumption 3.2, it shows that

$$d_{TV}(\mathbb{P}(\widetilde{s}_t = \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mu_{\theta_{t-\tau}}) \le m\rho^{\tau-1}.$$

Then we have

$$\mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau})] = \mathbb{E}[\Phi(\widetilde{O}_t, \eta_{t-\tau}, \theta_{t-\tau}) - \Phi(O'_{t-\tau}, \eta_{t-\tau}, \theta_{t-\tau})] \\ = \mathbb{E}[(\eta_{t-\tau} - J(\theta_{t-\tau}))(r(\widetilde{s}_t, \widetilde{a}_t) - r(s'_{t-\tau}, a'_{t-\tau}))] \\ \leq 4U_r^2 d_{TV}(\mathbb{P}(\widetilde{O}_{t-\tau} = \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mu_{\theta_{t-\tau}} \otimes \pi_{\theta_{t-\tau}} \otimes \mathcal{P}) \\ < 4U_r^2 m \rho^{\tau-1}.$$

Combing (27), (28), (29), and (32), we have

$$\begin{split} \mathbb{E}[\Phi(O_t,\eta_t,\theta_t)] &= \mathbb{E}[\Phi(O_t,\eta_t,\theta_t) - \Phi(O_t,\eta_t,\theta_{t-\tau})] + \mathbb{E}[\Phi(O_t,\eta_t,\theta_{t-\tau}) - \Phi(O_t,\eta_{t-\tau},\theta_{t-\tau})] \\ &+ \mathbb{E}[\Phi(O_t,\eta_{t-\tau},\theta_{t-\tau}) - \Phi(\widetilde{O}_t,\eta_{t-\tau},\theta_{t-\tau})] + \mathbb{E}[\Phi(\widetilde{O}_t,\eta_{t-\tau},\theta_{t-\tau})] \\ &\leq 4U_r L_J \|\theta_t - \theta_{t-\tau}\| + 2U_r |\eta_t - \eta_{t-\tau}| + 2U_r^2 |\mathcal{A}| L_\pi \sum_{i=t-\tau}^t \mathbb{E}\|\theta_i - \theta_{t-\tau}\| + 4U_r^2 m \rho^{\tau-1}, \end{split}$$

which concludes the proof.

#### Proof of Lemma C.3.

*Proof.* We will divide the proof of this lemma into four steps.

**Step 1:** show that for any  $\theta_1, \theta_2, \omega$  and tuple O = (s, a, s'), we have

$$\Psi(O,\omega,\theta_1) - \Psi(O,\omega,\theta_2) \le C_1 \|\theta_1 - \theta_2\|,\tag{33}$$

where  $C_1 = 2U_{\delta}^2 |\mathcal{A}| L_{\pi} (1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) + 2U_{\delta}L_J + 2U_{\delta}L_*.$ By definition of  $\Psi(O, \omega, \theta)$  in (17), we have

$$\begin{split} |\Psi(O,\omega,\theta_1) - \Psi(O,\omega,\theta_2)| &= |\langle \omega - \omega_1^*, g(O,\omega,\theta_1) - \bar{g}(\omega,\theta_1) \rangle - \langle \omega - \omega_2^*, g(O,\omega,\theta_2) - \bar{g}(\omega,\theta_2) \rangle| \\ &\leq \underbrace{|\langle \omega - \omega_1^*, g(O,\omega,\theta_1) - \bar{g}(\omega,\theta_1) \rangle - \langle \omega - \omega_1^*, g(O,\omega,\theta_2) - \bar{g}(\omega,\theta_2) \rangle|}_{I_1} \\ &+ \underbrace{|\langle \omega - \omega_1^*, g(O,\omega,\theta_2) - \bar{g}(\omega,\theta_2) \rangle - \langle \omega - \omega_2^*, g(O,\omega,\theta_2) - \bar{g}(\omega,\theta_2) \rangle|}_{I_2}. \end{split}$$

For term  $I_1$ , we have

$$\begin{split} I_1 &= |\langle \omega - \omega_1^*, g(O, \omega, \theta_1) - \bar{g}(\omega, \theta_1) \rangle - \langle \omega - \omega_1^*, g(O, \omega, \theta_2) - \bar{g}(\omega, \theta_2) \rangle| \\ &= |\langle \omega - \omega_1^*, g(O, \omega, \theta_1) - g(O, \omega, \theta_2) \rangle| + |\langle \omega - \omega_1^*, \bar{g}(\omega, \theta_1) - \bar{g}(\omega, \theta_2) \rangle| \\ &= |\langle \omega - \omega_1^*, \phi(s)(J(\theta_1) - J(\theta_2)) \rangle| + |\langle \omega - \omega_1^*, \bar{g}(\omega, \theta_1) - \bar{g}(\omega, \theta_2) \rangle| \\ &\leq 2U_\omega L_J \|\theta_1 - \theta_2\| + 2U_\omega \|\bar{g}(\omega, \theta_1) - \bar{g}(\omega, \theta_2)\| \\ &\leq 2U_\omega L_J \|\theta_1 - \theta_2\| + 2U_\omega \cdot 2U_\delta d_{TV} (\mu_{\theta_1} \otimes \pi_{\theta_1} \otimes \mathcal{P}, \mu_{\theta_2} \otimes \pi_{\theta_2} \otimes \mathcal{P}) \\ &\leq 2U_\omega L_J \|\theta_1 - \theta_2\| + 2U_\delta^2 d_{TV} (\mu_{\theta_1} \otimes \pi_{\theta_1} \otimes \mathcal{P}, \mu_{\theta_2} \otimes \pi_{\theta_2} \otimes \mathcal{P}) \\ &\leq (2U_\delta L_J + 2U_\delta^2 |\mathcal{A}| L_\pi (1 + \lceil \log_\rho m^{-1} \rceil + \frac{1}{1 - \rho}) \|\theta_1 - \theta_2\|, \end{split}$$

where we use the fact that  $U_{\delta} = 2U_r + 2U_{\omega}$  and the last inequality comes from Lemma B.4. For term  $I_2$ , from Cauchy-Schwartz inequality, we have

$$\begin{split} I_2 &= |\langle \omega - \omega_1^*, g(O, \omega, \theta_2) - \bar{g}(\omega, \theta_2) \rangle - \langle \omega - \omega_2^*, g(O, \omega, \theta_2) - \bar{g}(\omega, \theta_2) \rangle| \\ &= |\langle \omega_1^* - \omega_2^*, g(O, \omega, \theta_2) - \bar{g}(\omega, \theta_2) \rangle| \\ &\leq 2U_{\delta} \|\omega_1^* - \omega_2^* \| \\ &\leq 2U_{\delta} L_* \|\theta_1 - \theta_2 \|. \end{split}$$

Combining the results from  $I_1$  and  $I_2$ , we get

$$|\Psi(O,\omega,\theta_1) - \Psi(O,\omega,\theta_2) \le C_1 \|\theta_1 - \theta_2\|_{\mathcal{H}}$$

where  $C_1 = 2U_{\delta}^2 |\mathcal{A}| L_{\pi} (1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) + 2U_{\delta} L_J + 2U_{\delta} L_*.$ 

**Step 2:** show that for any  $\theta, \omega_1, \omega_2$  and tuple O(s, a, s'), we have

$$|\Psi(O,\omega_1,\theta) - \Psi(O,\omega_2,\theta)| \le 6U_{\delta} \|\omega_1 - \omega_2\|.$$
(34)

By definition, we have

$$\begin{split} \Psi(O,\omega_{1},\theta) - \Psi(O,\omega_{2},\theta) &|= |\langle \omega_{1} - \omega^{*}, g(O,\omega_{1},\theta) - \bar{g}(\omega_{1},\theta) \rangle - \langle \omega_{2} - \omega^{*}, g(O,\omega_{2},\theta) - \bar{g}(\omega_{2},\theta) \rangle |\\ &\leq |\langle \omega_{1} - \omega^{*}, g(O,\omega_{1},\theta) - \bar{g}(\omega_{1},\theta) \rangle - \langle \omega_{1} - \omega^{*}, g(O,\omega_{2},\theta) - \bar{g}(\omega_{2},\theta) \rangle |\\ &+ |\langle \omega_{1} - \omega^{*}, g(O,\omega_{2},\theta) - \bar{g}(\omega_{2},\theta) \rangle - \langle \omega_{2} - \omega^{*}, g(O,\omega_{2},\theta) - \bar{g}(\omega_{2},\theta) \rangle |\\ &\leq 2U_{\omega} \|(g(O,\omega_{1}) - g(O,\omega_{2})) - (\bar{g}(\omega_{1},\theta) - \bar{g}(\omega_{2},\theta))\| + 2U_{\delta} \|\omega_{1} - \omega_{2} \|\\ &\leq 6U_{\delta} \|\omega_{1} - \omega_{2}\|, \end{split}$$

where the last inequality is due to  $\|g(O,\omega_1,\theta) - g(O,\omega_2,\theta)\| = |(\phi(s') - \phi(s))^\top (\omega_1 - \omega_2)| \le 2\|\omega_1 - \omega_2\|, \|\bar{g}(\omega_1,\theta) - \bar{g}(\omega_2,\theta)\| \le 2\|\omega_1 - \omega_2\|, \text{ and } 2U_\omega \le U_\delta.$ 

**Step 3:** show that for tuples  $O_t = (s_t, a_t, s_{t+1})$  and  $\widetilde{O}_t = (\widetilde{s}_t, \widetilde{a}_t, \widetilde{s}_{t+1})$ . Conditioning on  $s_{t-\tau+1}$  and  $\theta_{t-\tau}$ , we have

$$\mathbb{E}[\Psi(O_t, \omega_{t-\tau}, \theta_{t-\tau}) - \Psi(\widetilde{O}_t, \omega_{t-\tau}, \theta_{t-\tau})] \le U_{\delta}^2 |\mathcal{A}| L_{\pi} \sum_{k=t-\tau}^{c} \mathbb{E} \|\theta_k - \theta_{t-\tau}\|.$$
(35)

By the definition of total variation norm, we have

$$\begin{split} \mathbb{E}[\Psi(O_t, \omega_{t-\tau}, \theta_{t-\tau}) - \Psi(O_t, \omega_{t-\tau}, \theta_{t-\tau})] &\leq \mathbb{E}[\langle \omega_{t-\tau} - \omega_{t-\tau}^*, g(O_t, \omega_{t-\tau}, \theta_{t-\tau}) - g(O_t, \omega_{t-\tau}, \theta_{t-\tau}))] \\ &\leq 2U_{\delta}^2 d_{TV}(\mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{-\tau}), \mathbb{P}(\widetilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}))) \\ &\leq U_{\delta}^2 |\mathcal{A}| L_{\pi} \sum_{k=t-\tau}^t \mathbb{E}||\theta_k - \theta_{t-\tau}|| \\ &\leq U_{\delta}^2 |\mathcal{A}| L_{\pi} G\tau(\tau+1)\alpha_{t-\tau}, \end{split}$$

where the last inequality comes from (31).

**Step 4:** show that conditioning on  $s_{t-\tau+1}$  and  $\theta_{t-\tau}$ ,

$$\mathbb{E}[\Psi(\widetilde{O}_t, \omega_{t-\tau}, \theta_{t-\tau})] \le 2U_\delta^2 m \rho^{\tau-1}$$
(36)

From the definition of  $\Psi(O, \omega, \theta)$ , we have

$$\mathbb{E}[\Psi(O'_{t-\tau},\omega_{t-\tau},\theta_{t-\tau})|s_{t-\tau+1},\theta_{t-\tau}]=0,$$

where  $O'_{t-\tau}$  is the tuple generated by  $s'_{t-\tau} \sim \mu_{\theta_{t-\tau}}, a'_{t-\tau} \sim \pi_{\theta_{t-\tau}}, s'_{t-\tau+1} \sim \mathcal{P}$ . From Assumption 3.2, we have

$$d_{TV}(\mathbb{P}(\widetilde{s}_t = \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mu_{\theta_{t-\tau}}) \le m\rho^{\tau-1}.$$

Then, it holds that

$$\mathbb{E}[\Psi(\tilde{O}_{t},\omega_{t-\tau},\theta_{t-\tau})] = \mathbb{E}[\Psi(\tilde{O}_{t},\omega_{t-\tau},\theta_{t-\tau}) - \Psi(O'_{t-\tau},\omega_{t-\tau},\theta_{t-\tau})] \\ = \mathbb{E}\langle\omega_{t-\tau} - \omega^{*}_{t-\tau},g(\tilde{O}_{t},\omega_{t-\tau},\theta_{t-\tau} - g(O'_{t-\tau},\omega_{t-\tau},\theta_{t-\tau})\rangle \\ \leq 4U_{\omega}U_{\delta}d_{TV}(\mathbb{P}(\tilde{O}_{t}=\cdot|s_{t-\tau+1},\theta_{t-\tau}),\mu_{\theta_{t-\tau}}\otimes\pi_{\theta_{t-\tau}}\otimes\mathcal{P}) \\ \leq 2U^{2}_{\delta}d_{TV}(\mathbb{P}(\tilde{O}_{t}=\cdot|s_{t-\tau+1},\theta_{t-\tau}),\mu_{\theta_{t-\tau}}\otimes\pi_{\theta_{t-\tau}}\otimes\mathcal{P}) \\ = 2U^{2}_{\delta}d_{TV}(\mathbb{P}((\tilde{s}_{t},\tilde{a}_{t})\in\cdot|s_{t-\tau+1},\theta_{t-\tau}),\mu_{\theta_{t-\tau}}\otimes\pi_{\theta_{t-\tau}}) \\ = 2U^{2}_{\delta}d_{TV}(\mathbb{P}(\tilde{s}_{t}=\cdot|s_{t-\tau+1},\theta_{t-\tau}),\mu_{\theta_{t-\tau}}) \\ \leq 2U^{2}_{\delta}m\rho^{\tau-1}.$$

Combining (33), (34), (35), and (36), we have

$$\mathbb{E}[\Psi(O_t,\omega_t,\theta_t)] = \mathbb{E}[\Psi(O_t,\omega_t,\theta_t) - \Psi(O_t,\omega_t,\theta_{t-\tau})] + \mathbb{E}[\Psi(O_t,\omega_t,\theta_{t-\tau}) - \Psi(O_t,\omega_{t-\tau},\theta_{t-\tau})] \\ + \mathbb{E}[\Psi(O_t,\omega_{t-\tau},\theta_{t-\tau}) - \Psi(\widetilde{O}_t,\omega_{t-\tau},\theta_{t-\tau})] + \mathbb{E}[\Psi(\widetilde{O}_t,\omega_{t-\tau},\theta_{t-\tau})] \\ \leq C_1 \|\theta_t - \theta_{t-\tau}\| + U_{\delta}^2 |\mathcal{A}| L_{\pi} G\tau(\tau+1)\alpha_{t-\tau} + 2U_{\delta}^2 m \rho^{\tau-1} + 6U_{\delta} \|\omega_t - \omega_{t-\tau}\|,$$

where  $C_1 = 2U_{\delta}^2 |\mathcal{A}| L_{\pi} (1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1-\rho}) + 2U_{\delta} (L_J + L_*).$ 

# Proof of Lemma C.5.

*Proof.* We will divide the proof of this lemma into three steps. **Step 1:** show that

$$|\Theta(O_t, \theta_{t-\tau}) - \Theta(\widetilde{O}_t, \theta_{t-\tau})| \le (2U_\delta B L_{J'} + 3L_J L_h) \|\theta_t - \theta_{t-\tau}\|,\tag{37}$$

where  $L_h = U_{\delta}L_l + (2 + 2\lambda^{-2} + 3\lambda^{-1})BU_r |\mathcal{A}|L_{\pi}(1 + \lceil \log_{\rho} m^{-1} \rceil + 1/(1 - \rho)).$ Since  $\Theta(O, \theta) = \langle \nabla J(\theta), \mathbb{E}_{\theta}[h(O', \theta)] - h(O, \theta) \rangle$ , we will show that each term in  $\Theta(O, \theta)$  is Lipschitz. For the term  $\nabla J(\theta)$ , by Lemma B.3 we know it's  $L_{J'}$ -Lipschitz. For term  $h(O, \theta)$ , denote  $\delta(O, \theta) := r(s, a) - r(\theta) + (\phi(s') - \phi(s))^{\top} \omega^*$ , we have  $\|h(O, \theta_1) - h(O, \theta_2)\| = \|\delta(O, \theta_1) \nabla \log \pi_{\theta_1}(a|s) - \delta(O_t, \theta_2) \nabla \log \pi_{\theta_2}(a|s)\|$   $\leq \|\delta(O, \theta_1) \nabla \log \pi_{\theta_1}(a|s) - \delta(O, \theta_1) \nabla \log \pi_{\theta_2}(a|s)\|$   $+ \|\delta(O, \theta_1) \nabla \log \pi_{\theta_2}(a|s) - \delta(O, \theta_2) \nabla \log \pi_{\theta_2}(a|s)\|$   $\leq U_{\delta}L_l \|\theta_1 - \theta_2\| + B|\delta(O, \theta_1) - \delta(O, \theta_2)|$   $\leq U_{\delta}L_l \|\theta_1 - \theta_2\| + B(|r(\theta_1) - r(\theta_2)| + \|\phi(s') - \phi(s)\| \cdot \|\omega^*(\theta_1) - \omega^*(\theta_2)\|)$   $\leq (U_{\delta}L_l + 2BL_*) \|\theta_1 - \theta_2\| + B|\mathbb{E}_{s \sim \mu_{\theta_1}, a \sim \pi_{\theta_1}}[r(s, a)] - \mathbb{E}_{s \sim \mu_{\theta_1}, a \sim \pi_{\theta_2}}[r(s, a)]|$   $\leq (U_{\delta}L_l + 2BL_*) \|\theta_1 - \theta_2\| + 2BU_r d_{TV}(\mu_{\theta_1} \otimes \pi_{\theta_1}, \mu_{\theta_2} \otimes \pi_{\theta_2})$  $\leq (U_{\delta}L_l + 2BL_* + 2BU_r |\mathcal{A}|L_{\pi}(1 + \lceil \log_{\rho} m^{-1} \rceil + \frac{1}{1 - \rho})) \|\theta_1 - \theta_2\|.$ 

Hence we have  $h(O, \theta)$  is  $L_h$ -Lipschitz, where  $L_h$  denotes the above coefficient. For term  $\mathbb{E}_{\theta}[h(O', \theta)]$ , we have

$$\begin{split} \|\mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{1})] - \mathbb{E}_{\theta_{2}}[h(O',\theta_{2})]\| &\leq \|\mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{1})] - \mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{2})]\| + \|\mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{2})] - \mathbb{E}_{\theta_{2}}[h(O_{t},\theta_{2})]\| \\ &\leq \mathbb{E}_{\theta_{1}}[\|h(O',\theta_{1}) - h(O',\theta_{2})\|] + \|\mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{2})] - \mathbb{E}_{\theta_{2}}[h(O_{t},\theta_{2})]\| \\ &\leq L_{h}\|\theta_{1} - \theta_{2}\| + \|\mathbb{E}_{\theta_{1}}[h(O_{t},\theta_{2})] - \mathbb{E}_{\theta_{2}}[h(O_{t},\theta_{2})]\| \\ &\leq L_{h}\|\theta_{1} - \theta_{2}\| + 2BU_{r}d_{TV}(\mu_{\theta_{1}} \otimes \pi_{\theta_{1}},\mu_{\theta_{2}} \otimes \pi_{\theta_{2}}) \\ &\leq [L_{h} + 2BU_{r}|\mathcal{A}|L_{\pi}(1 + \lceil\log_{\rho}m^{-1}\rceil + \frac{1}{1-\rho})]\|\theta_{1} - \theta_{2}\| \\ &\leq 2L_{h}\|\theta_{1} - \theta_{2}\|. \end{split}$$

Then we have  $\nabla J(\theta)$  is  $L_J$ -bounded and  $L_{J'}$ -Lipschitz;  $h(O, \theta) - \mathbb{E}_{\theta}[h(O', \theta)]$  is  $3L_h$ -Lipschitz and  $2U_{\delta}B$ -bounded. By the triangle inequality, we have

$$\Theta(O_t, \theta_t) - \Theta(O_t, \theta_{t-\tau}) \le (2U_{\delta}BL_{J'} + 3L_JL_h) \|\theta_t - \theta_{t-\tau}\|$$

**Step 2:** show that for  $t \ge \tau > 0$ , we have

$$|\mathbb{E}[\Theta(O_t, \theta_{t-\tau}) - \Theta(\widetilde{O}_t, \theta_{t-\tau})]| \le 2U_{\delta}BL_J |\mathcal{A}| L_{\pi} \sum_{k=t-\tau}^t \|\theta_k - \theta_{t-\tau}\|$$
(38)

By definition of  $\Theta(O, \theta)$ , we have

$$\begin{aligned} |\mathbb{E}[\Theta(O_t, \theta_{t-\tau}) - \Theta(\widetilde{O}_t, \theta_{t-\tau})]| &= |\mathbb{E}[\langle \nabla J(\theta_{t-\tau}), h(\widetilde{O}_t, \theta_{t-\tau}) - h(O_t, \theta_{t-\tau})]| \\ &= |\mathbb{E}[\langle \nabla J(\theta_{t-\tau}), h(\widetilde{O}_t, \theta_{t-\tau}) \rangle - \mathbb{E}[\langle \nabla J(\theta_{t-\tau}), h(O_t, \theta_{t-\tau}) \rangle]| \\ &\leq 4U_{\delta}BL_J d_{TV}(\mathbb{P}(O_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mathbb{P}(\widetilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau})), \end{aligned}$$
(39)

where the inequality comes from the definition of total variation distance. The total variation norm between  $O_t$  and  $\tilde{O}_t$  has been computed in (31). Plugging (31) into (39), we get

$$|\mathbb{E}[\Theta(O_t, \theta_{t-\tau}) - \Theta(\widetilde{O}_t, \theta_{t-\tau})]| \le 2U_{\delta}BL_J |\mathcal{A}| L_{\pi} \sum_{k=t-\tau}^t \|\theta_k - \theta_{t-\tau}\|.$$

**Step 3:** show that for  $t \ge \tau > 0$ , we have

$$|\mathbb{E}[\Theta(\widetilde{O}_t, \theta_{t-\tau} - \Theta(O'_t, \theta_{t-\tau})]| \le 4U_{\delta}BL_J m \rho^{\tau-1}.$$
(40)

From the definition of  $\Theta(O, \theta)$ , we have

$$\begin{aligned} |\mathbb{E}[\Theta(\tilde{O}_t, \theta_{t-\tau}) - \Theta(O'_t, \theta_{t-\tau})]| &= |\mathbb{E}[\langle \nabla J(\theta_{t-\tau}), h(O'_t, \theta_{t-\tau}) \rangle - \langle \nabla J(\theta_{t-\tau}), h(\tilde{O}_t, \theta_{t-\tau}) \rangle]| \\ &\leq 4U_{\delta}BL_J d_{TV}(\mathbb{P}(\tilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}), \mu_{\theta_{t-\tau}} \otimes \pi_{\theta_{t-\tau}} \otimes \mathcal{P}). \end{aligned}$$

The inequality is due to the definition of total variation distance. From Assumption 3.2, we know that

 $d_{TV}(\mathbb{P}(\widetilde{s}_t \in \cdot), \mu_{\theta_{t-\tau}}) \le m \rho^{\tau-1}.$ 

We also have the fact that

$$\mathbb{P}(\widetilde{O}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}) = \mathbb{P}(\widetilde{s}_t \in \cdot | s_{t-\tau+1}, \theta_{t-\tau}) \otimes \pi_{\theta_{t-\tau}} \otimes \mathcal{P}.$$

Therefore, we have

$$|\mathbb{E}[\Theta(\widetilde{O}_t, \theta_{t-\tau} - \Theta(O'_t, \theta_{t-\tau})]| \le 4U_{\delta}BL_J m \rho^{\tau-1}$$

Combining (37), (38), and (40), we can decompose the Markovian bias as

$$\begin{split} \mathbb{E}[\Theta(O_t,\theta_t)] &= \mathbb{E}[\Theta(O_t,\theta_t) - \Theta(O_t,\theta_{t-\tau})] + \mathbb{E}[\Theta(O_t,\theta_{t-\tau}) - \Theta(O_t,\theta_{t-\tau})] \\ &+ \mathbb{E}[\Theta(\widetilde{O}_t,\theta_{t-\tau}) - \Theta(O_t',\theta_{t-\tau})] + \mathbb{E}[\Theta(O_t',\theta_{t-\tau})], \end{split}$$

where  $\tilde{O}_t$  is from the auxiliary Markovian chain defined in (14) and  $O'_t$  is from the stationary distribution which satisfies  $\mathbb{E}[\Theta(O'_t, \theta_{t-\tau})] = 0$ .

Then we have

$$\begin{split} \mathbb{E}[\Theta(O_{t},\theta_{t})] &\leq (2U_{\delta}BL_{J'} + 3L_{J}L_{h})\mathbb{E}\|\theta_{t} - \theta_{t-\tau}\| + 2U_{\delta}BL_{J}|\mathcal{A}|L_{\pi}\sum_{k=t-\tau}^{t}\|\theta_{k} - \theta_{t-\tau}\| + 4U_{\delta}BL_{J}m\rho^{\tau-1} \\ &\leq (2U_{\delta}BL_{J'} + 3L_{J}L_{h})\sum_{k=t-\tau+1}^{t}\mathbb{E}\|\theta_{k} - \theta_{k-1}\| + 2U_{\delta}BL_{J}|\mathcal{A}|L_{\pi}\sum_{k=t-\tau+1}^{t}\sum_{j=t-\tau+1}^{k}\mathbb{E}\|\theta_{j} - \theta_{j-1}\| \\ &+ 4U_{\delta}BL_{J}m\rho^{\tau-1} \\ &\leq (2U_{\delta}BL_{J'} + 3L_{J}L_{h})\sum_{k=t-\tau+1}^{t}\mathbb{E}\|\theta_{k} - \theta_{k-1}\| + 2U_{\delta}BL_{J}|\mathcal{A}|L_{\pi}\tau\sum_{j=t-\tau+1}^{t}\mathbb{E}\|\theta_{j} - \theta_{j-1}\| \\ &+ 4U_{\delta}BL_{J}m\rho^{\tau-1} \\ &\leq D_{1}(\tau+1)\sum_{k=t-\tau+1}^{t}\mathbb{E}\|\theta_{k} - \theta_{k-1}\| + D_{2}m\rho^{\tau-1}, \end{split}$$

where  $D_1 = \max\{U_{\delta}BL_{J'} + 3L_JL_h, 2U_{\delta}BL_J|\mathcal{A}|L_{\pi}\}$  and  $D_2 = 4U_{\delta}BL_J$ . Thus we conclude the proof.

# **E IID** Sampling Analysis

Algorithm 2 Single-timescale Actor-Critic (i.i.d. sampling)

1: Input initial actor parameter  $\theta_0$ , initial critic parameter  $\omega_0$ , initial reward estimator  $\eta_0$ , stepsize  $\alpha_t$  for actor,  $\beta_t$ for critic and  $\gamma_t$  for reward estimator. 2: for  $t = 0, 1, 2, \dots, T - 1$  do 3: Sample  $s_t \sim \mu_{\theta_t}$ 4: Take the action  $a_t \sim \pi_{\theta_t}(\cdot|s_t)$ 5: Observe next state  $s'_t \sim \mathcal{P}(\cdot|s_t, a_t)$  and the reward  $r_t = r(s_t, a_t)$ 6:  $\delta_t = r_t - \eta_t + \phi(s'_t)^\top \omega_t - \phi(s_t)^\top \omega_t$ 7:  $\eta_{t+1} = \eta_t + \gamma_t(r_t - \eta_t)$ 8:  $\omega_{t+1} = \prod_{U_\omega} (\omega_t + \beta_t \delta_t \phi(s_t))$ 9:  $\theta_{t+1} = \theta_t + \alpha_t \delta_t \nabla_{\theta} \log \pi_{\theta_t}(a_t|s_t)$ 10: end for

Note that under i.i.d. sampling in Algorithm 2, we denote by  $s_t$  the samples from the stationary distribution and  $s'_t$  the subsequent state following transition kernel  $s'_t \sim \mathcal{P}(\cdot|s_t, a_t)$ . Correspondingly, we redefine the observation tuple as  $O_t = (s_t, a_t, s'_t)$  (in contrast to  $O_t = (s_t, a_t, s_{t+1})$  in the Markovian sampling case). This modification implies the decoupling of  $O_t$  and  $O_{t+1}$  since  $s_{t+1}$  in tuple  $O_{t+1}$  is a new state sampled from the stationary distribution rather than inherited from  $O_t$ . This intuitively elucidates the vanishment of Markovian noise under i.i.d. sampling.

Lemma E.1. Under i.i.d sampling, we have

$$\begin{split} \mathbb{E}[\Phi(O_t, \eta_t, \theta_t)] &= 0, \\ \mathbb{E}[\Psi(O_t, \omega_t, \theta_t)] &= 0, \\ \mathbb{E}[\Theta(O_t, O'_t, \theta_t)] &= 0. \end{split}$$

*Proof.* Note that the expectation is taken over all the random variables. We use the notation  $O_t$  to denote the tuple  $(s_t, a_t, s'_t)$  and  $v_{0:t}$  to denote the sequence  $(s_t, a_t, s'_t), (s_t, a_t, s'_t), \cdots, (s_t, a_t, s'_t)$ . By definition in (17), it can be shown that

$$\begin{split} \mathbb{E}[\Phi(O_t, \eta_t, \theta_t)] &= \mathbb{E}_{v_{0:t}}[\Phi(O_t, \eta_t, \theta_t)] \\ &= \mathbb{E}_{v_{0:t-1}} \mathbb{E}_{v_{0:t}}[(\eta_t - J(\theta_t))(r_t - J(K_t))|v_{0:t-1}], \end{split}$$

where is second equality is due to law of total expectation. Once we know  $v_{0:t-1}$ ,  $\eta_t$  and  $J(\theta_t)$  is not a random variable any more. It holds that

$$\mathbb{E}[\Phi(O_t, \eta_t, \theta_t)] = \mathbb{E}_{v_{0:t-1}} \mathbb{E}_{v_{0:t}}[(\eta_t - J(\theta_t))(r_t - J(K_t))|v_{0:t-1}] \\ = \mathbb{E}_{v_{0:t-1}}(\eta_t - J(\theta_t))\mathbb{E}_{v_{0:t}}[(r_t - J(K_t))|v_{0:t-1}] \\ = \mathbb{E}_{v_{0:t-1}}(\eta_t - J(\theta_t))\mathbb{E}_{O_t}[(r_t - J(K_t))|v_{0:t-1}] \\ = 0,$$

where the last equation is due to  $\mathbb{E}_{O_t}[(r_t - J(K_t))|v_{0:t-1}] = 0$  under i.i.d. sampling. By the similar argument, we have

$$\begin{split} \mathbb{E}[\Psi(O_t, \eta_t, \theta_t)] &= \mathbb{E}_{v_{0:t}}[\langle \omega_t - \omega_t^*, g(O, \omega, \theta) - \bar{g}(\omega_t, \theta_t) \rangle] \\ &= \mathbb{E}_{v_{0:t-1}} \mathbb{E}_{v_{0:t}}[\langle \omega_t - \omega_t^*, g(O_t, \omega_t, \theta_t) - \bar{g}(\omega_t, \theta_t) \rangle | v_{0:t-1}] \\ &= \mathbb{E}_{v_{0:t-1}} \langle \omega_t - \omega_t^*, \mathbb{E}_{v_{0:t}}[g(O_t, \omega_t, \theta_t) - \bar{g}(\omega_t, \theta_t) \rangle | v_{0:t-1}] \\ &= \mathbb{E}_{v_{0:t-1}} \langle \omega_t - \omega_t^*, \mathbb{E}_{O_t}[g(O_t, \omega_t, \theta_t) - \bar{g}(\omega_t, \theta_t) \rangle | v_{0:t-1}] \\ &= 0, \end{split}$$

where we use the fact that  $\mathbb{E}_{O_t}[g(O_t, \omega_t, \theta_t) - \bar{g}(\omega_t, \theta_t)\rangle |v_{0:t-1}] = 0.$ 

Similarly, we have

$$\begin{split} \mathbb{E}[\Theta(O_t, O'_t, \theta_t)] &= \mathbb{E}_{v_{0:t}}[\langle \nabla J(\theta_t), \mathbb{E}_{O'_t}[h(O'_t, \theta_t)] - h(O_t, \theta_t)\rangle] \\ &= \mathbb{E}_{v_{0:t-1}} \mathbb{E}_{v_{0:t}}[\langle \nabla J(\theta_t), \mathbb{E}_{O'_t}[h(O'_t, \theta_t)] - h(O_t, \theta_t)\rangle|v_{0:t-1}] \\ &= \mathbb{E}_{v_{0:t-1}}\langle \nabla J(\theta_t), \mathbb{E}_{v_{0:t}}[\mathbb{E}_{O'_t}[h(O'_t, \theta_t)] - h(O_t, \theta_t)\rangle|v_{0:t-1}] \\ &= \mathbb{E}_{v_{0:t-1}}\langle \nabla J(\theta_t), \mathbb{E}_{O_t}[\mathbb{E}_{O'_t}[h(O'_t, \theta_t)] - h(O_t, \theta_t)\rangle|v_{0:t-1}] \\ &= 0, \end{split}$$

where we use fact that  $O_t = O'_t$  under i.i.d. sampling.

#### **Proof of Theorem 3.5**.

*Proof.* The proof follows similarly to the Markovian sampling case. Specifically, all the Markovian noises (see the definitions in (17)) present in the former analysis reduce to zero after taking expectations. The detailed results and proof are presented in Lemma E.1. Then, replacing Lemma C.1, Lemma C.3, and Lemma C.5 with Lemma E.1, we will get the desired  $\mathcal{O}(T^{-\frac{1}{2}})$  convergence rate and thus an  $\mathcal{O}(\epsilon^{-2})$  sample complexity accordingly.